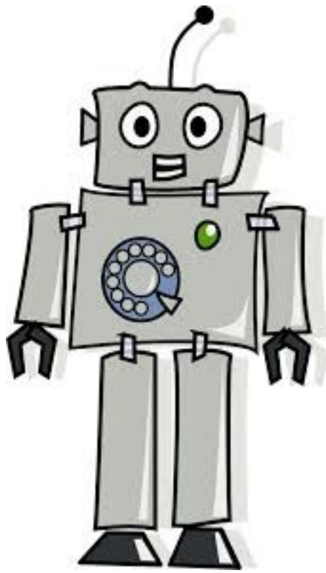
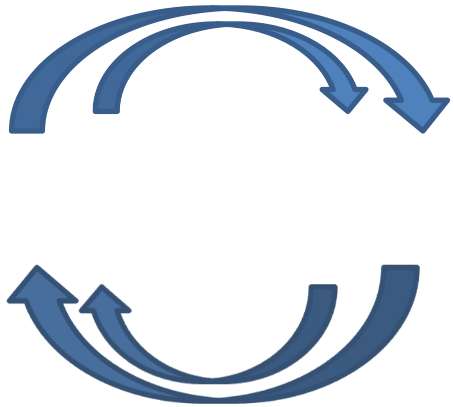
CMSC 478: Reinforcement Learning

Some slides courtesy Cynthia Matuszek and Frank Farrero, with some material from Marie desJardin, Lise Getoor, Jean-Claude Latombe, and Daphne Koller

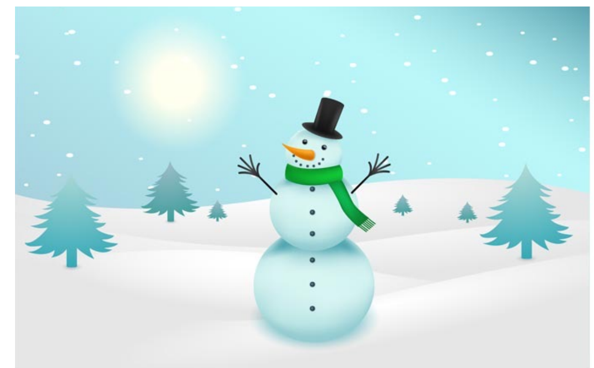
Markov Decision Process: Formalizing Reinforcement Learning

take action





get new state
and/or reward



environment

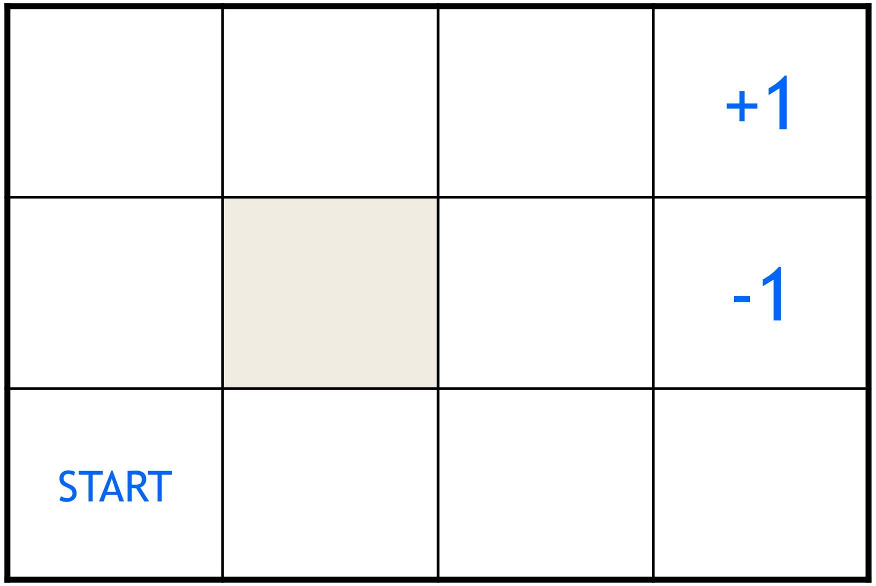
agent

Markov Decision Process: $(\mathcal{S}, \mathcal{A}, \mathcal{R}, P, \gamma)$

Robot image: openclipart.org

https://static.vecteezy.com/system/resources/previews/000/090/451/original/four-seasons-landscape-illustrations-vector.jpg

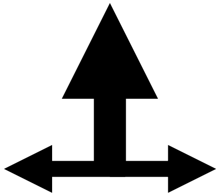
Robot in a room



actions: UP, DOWN, LEFT, RIGHT

UP

80% move UP
10% move LEFT
10% move RIGHT

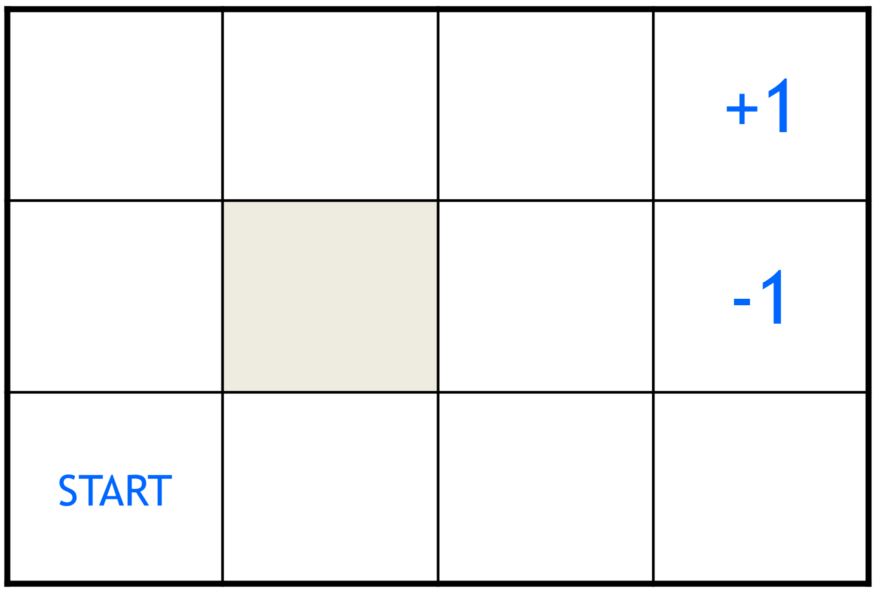


reward +1 at [4,3], -1 at [4,2]
reward -0.04 for each step

Goal: what's the strategy to achieve the maximum reward?

Slide courtesy Peter Bodík

Robot in a room



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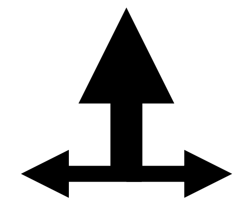


reward +1 at [4,3], -1 at [4,2]
reward -0.04 for each step

states: current location
actions: where to go next
rewards

what is the solution? Learn a mapping from (state, action) pairs to new states

Slide courtesy Peter Bodík

Markov Decision Process: Formalizing Reinforcement Learning

Markov Decision Process: $(\mathcal{S}, \mathcal{A}, \mathcal{R}, P, \gamma)$

set of possible states

set of possible actions

reward of (state, action) pairs

state-action transition distribution

discount factor

Start in initial state s_0
for t = 1 to …:
  choose action a_t
  "move" to next state $s_t \sim \pi(\cdot|s_{t-1}, a_t)$
  get reward $r_t = \mathcal{R}(s_t, a_t)$

objective: maximize discounted reward

$$\max_{\pi} \sum_{t>0} \gamma^t r_t$$

"solution" $\pi^* = \underset{\pi}{\operatorname{argmax}}\, \mathbb{E}\left[\sum_{t>0} \gamma^t r_t\,; \pi\right]$

Overview: Learning Strategies

Dynamic Programming

Q-learning

Monte Carlo approaches

Dynamic programming

use value functions to structure the search for good policies

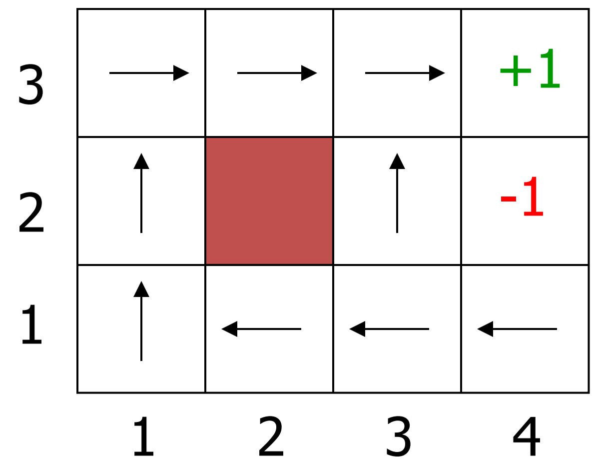
policy evaluation: compute V^{π} from π

policy improvement: improve π based on V^{π}

start with an arbitrary policy

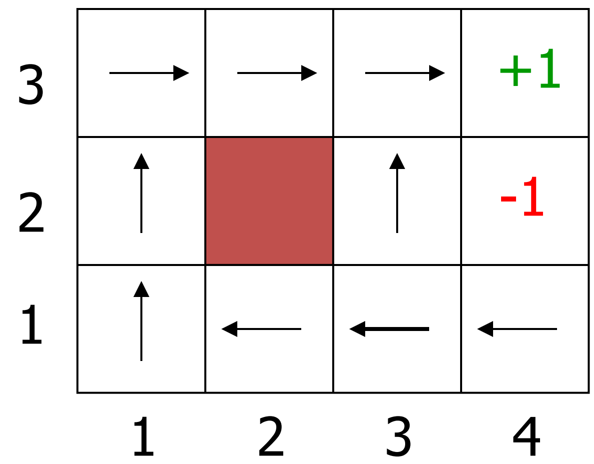
repeat evaluation/improvement until convergence

Optimal Policy



- A policy Π is a complete mapping from states to actions
- The optimal policy Π* is the one that always yields a history (sequence of steps ending at a terminal state) with maximal ***expected*** utility

Optimal Policy



- A policy Π is a comp… ns
- The optimal policy Π… history with maximal expected utility

This problem is called a Markov Decision Problem (MDP)

How to compute Π*?

Defining Value Function

- Problem:
  - When making a decision, we only know the reward so far, and the possible actions
  - We've defined value function retroactively (i.e., the value function/utility of a history/sequence of states is known *once we finish it*)
  - What is the value function of a particular ***state*** in the middle of decision making?
  - Need to compute ***expected value function*** of possible future histories/states

Defining Value Function

$$V^{\pi}(s) = \mathrm{E}\left[R(s_0) + \gamma R(s_1) + \gamma^2 R(s_2) + \cdots \,\middle|\, s_0 = s, \pi\right].$$

$V^{\pi}(s)$ is simply the expected sum of discounted rewards upon starting in state s, and taking actions according to π.[1]

Given a fixed policy π, its value function V^{π} satisfies the **Bellman equations**:

$$V^{\pi}(s) = R(s) + \gamma \sum_{s' \in S} P_{s\pi(s)}(s') V^{\pi}(s').$$

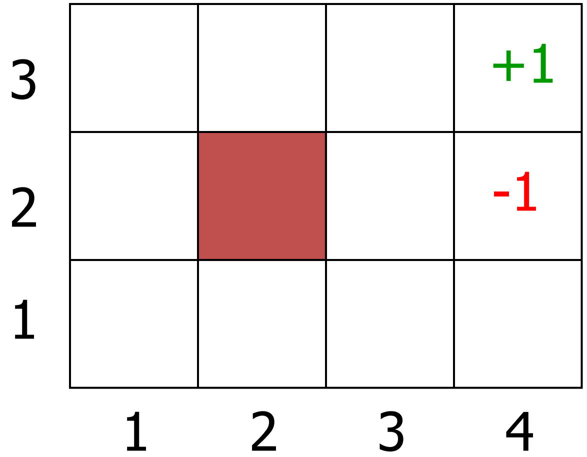
- What is the value function of a particular ***state*** in the middle of decision making?
- Need to compute ***expected value function*** of possible future histories/states

Value Iteration

Algorithm 4 Value Iteration

For each state s, initialize $V(s) := 0$.
for until convergence **do**
    For every state, update

$$V^*(s) = R(s) + \max_{a \in A} \gamma \sum_{s' \in S} P_{sa}(s') V^*(s'). \tag{15.4}$$



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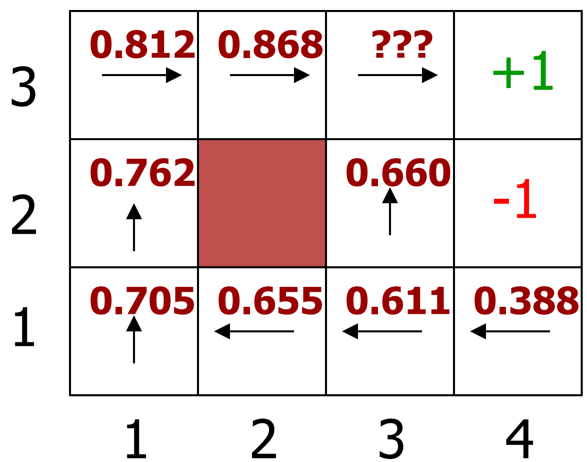
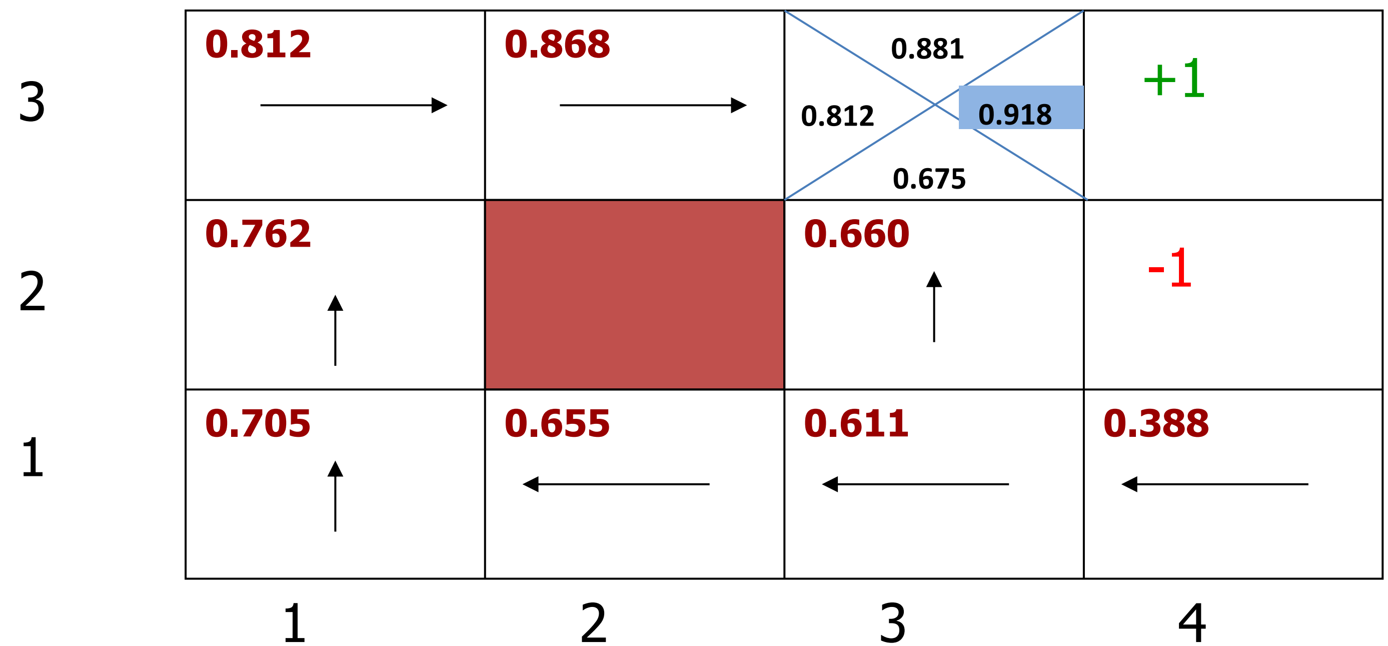


More Breakdown

Value Iteration

In (3, 3), since → action gave us the **maximum expected future reward,** we choose to keep → in our policy. Same thing was done for all states.



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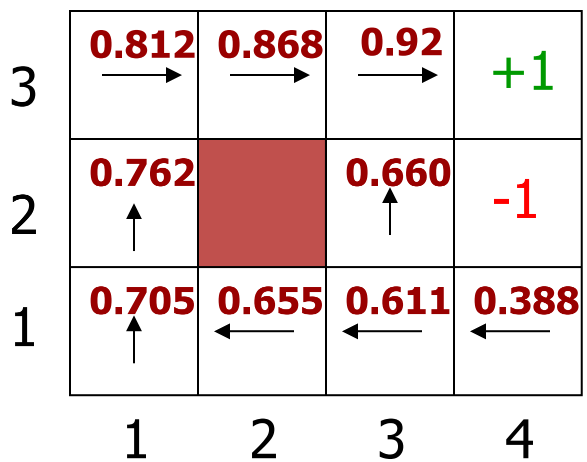

EXERCISE: What is V*([3,3]) (assuming that the other V* are as shown)?

Value Iteration

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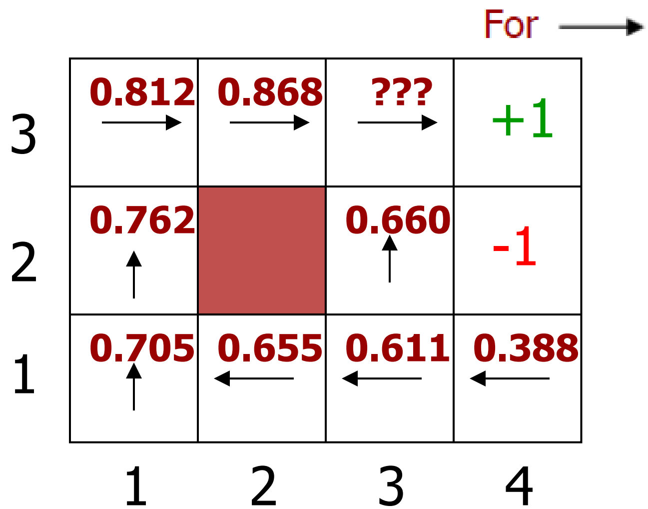

From (3, 3), 3 options: (3, 2), (4, 3), (3, 4) => but there is no (3,4) but wall, so bounced off and remains at (3, 3)

EXERCISE: What is **next** V*([3,3]) (assuming that other V* are as shown)?

Value Iteration

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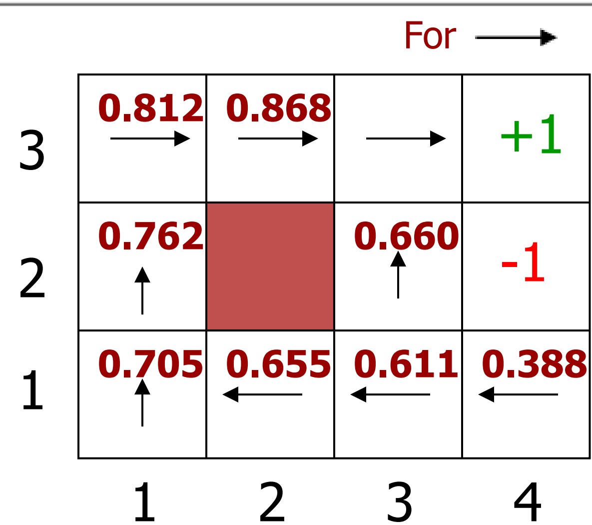

$V^*_{3,3} = R_{3,3} +$
$[P_{3,2}\, V^*_{3,2} + P_{3,3}\, V^*_{3,3} + P_{4,3}\, V^*_{4,3}]$

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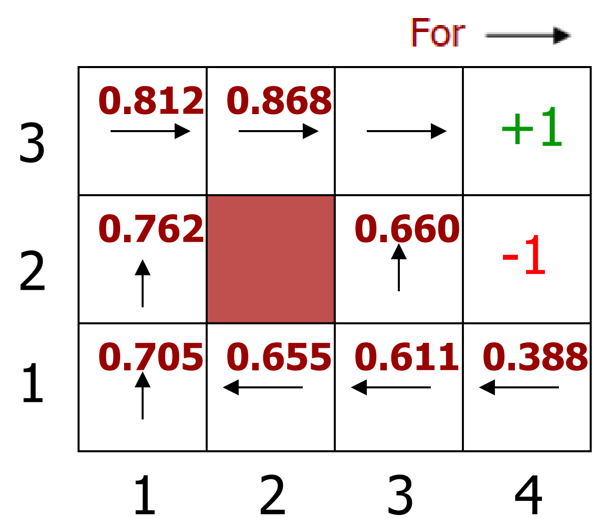

$V^*_{3,3} = R_{3,3} +$
$[P_{3,2}\ V^*_{3,2} + P_{3,3}\ V^*_{3,3} + P_{4,3}\ V^*_{4,3}]$
$= -0.04 +$
$[0.1*0.660 + 0.1*0.92 + 0.8*1]$

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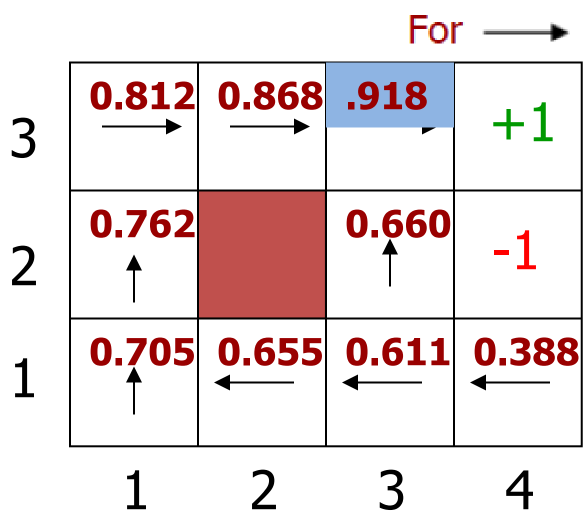

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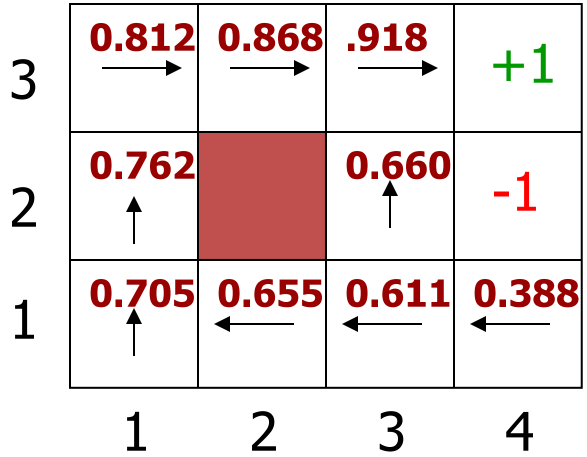
From (3, 3), 3 options: (3, 2), (4, 3), (3, 4) => but there is no (3,4) but wall, so bounced off and remains at (3, 3)

Optimal Policy

$$\pi^*(s) = \arg\max_{a \in A} \sum_{s' \in S} P_{sa}(s') V^*(s').$$



Whichever is higher becomes next action for (3, 1)

Optimal Policy

$$\pi^*(s) = \arg\max_{a \in A} \sum_{s' \in S} P_{sa}(s') V^*(s').$$

$\pi^*_{3,1}$ being $(\leftarrow)$ =

$P_{up}\ V^*_{2,1} + P_{left}\ V^*_{3,1}$ (Bounced off) $+ P_{right}\ V^*_{3,2}$

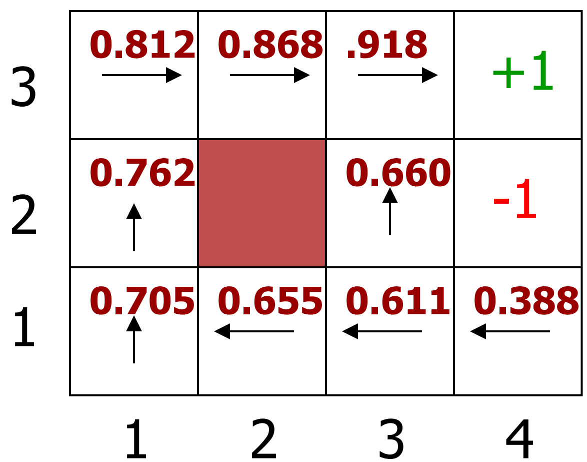
= 0.8 * 0.655 + 0.1 * 0.611 + 0.1 * 0.66

	1	2	3	4
3	0.812 →	0.868 →	.918 →	+1
2	0.762 ↑		0.660 ↑	-1
1	0.705 ↑	0.655 ←	0.611 ←	0.388 ←

Whichever is higher becomes next action for (3, 1)

Optimal Policy

$$\pi^*(s) = \arg\max_{a \in A} \sum_{s' \in S} P_{sa}(s') V^*(s').$$



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$= 0.8 * 0.655 + 0.1 * 0.611 + 0.1 * 0.66$

$\pi^*_{3,1}$ being ($\uparrow$) =

$P_{up}\ V^*_{3,2} + P_{left}\ V^*_{2,1} + P_{right}\ V^*_{1,4}$

Whichever is higher becomes
next action for (3, 1)

Policy Iteration

- Pick a policy Π at random
- Repeat:
  - Compute Value function of each state for Π

$$V(s) := V^{\pi}(s) = R(s) + \gamma \sum_{s' \in S} P_{s\pi(s)}(s') V^{\pi}(s').$$

  - Compute the policy Π' given these value functions

$$\pi'(s) := \arg\max_{a \in A} \sum_{s'} P_{sa}(s') V(s').$$

  - If $\Pi' = \Pi$ then return Π

Policy Iteration

- Pick a policy Π at random
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$$V(s) := V^{\pi}(s) = R(s) + \gamma \sum_{s' \in S} P_{s\pi(s)}(s') V^{\pi}(s').$$

  - Compute the policy Π' given these value functions

Or solve the set of linear equations:
(often a sparse system)

$$\pi'(s) := \arg\max_{a \in A} \sum_{s'} P_{sa}(s') V(s').$$

  - If $\Pi' = \Pi$ then return Π

Value Iteration: Summary

- Initialize state values (expected utilities) randomly
- Repeatedly update state values using best action, according to current approximation of state values
- Terminate when state values stabilize
- Resulting policy will be the best policy because it’s based on accurate state value estimation

Policy Iteration: Summary

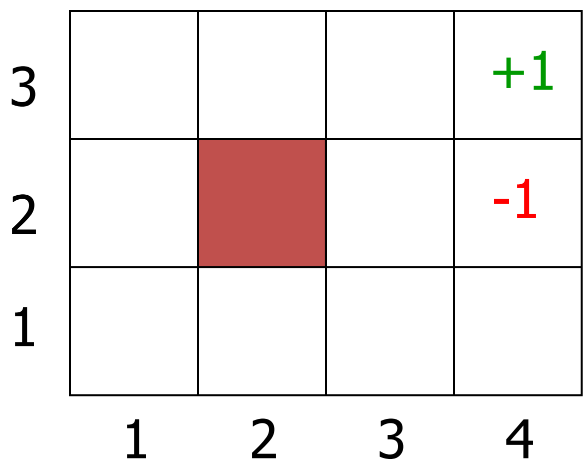
– Initialize policy randomly
– Repeatedly update state values using best action, according to current approximation of state values
– Then update policy based on new state values
– Terminate when policy stabilizes
– Resulting policy is the best policy, but state values may not be accurate (may not have converged yet)
– Policy iteration is often faster (because we don't have to get the state values right)

- **Both methods have a major weakness: They require us to know the transition function exactly in advance!**

Advanced topic

Infinite Horizon

In many problems, e.g., the robot navigation example, histories are potentially unbounded and the same state can be reached many times



Advanced topic

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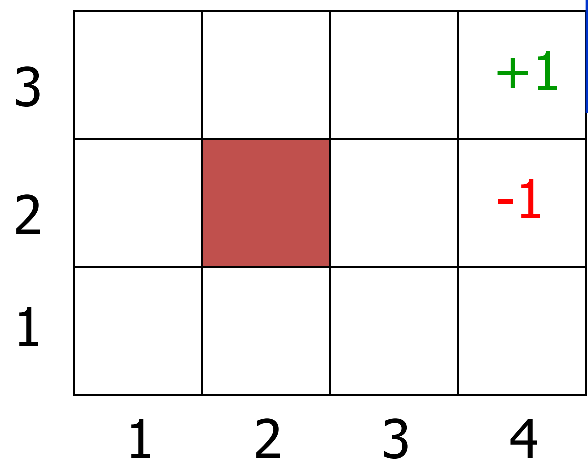

What if the robot lives forever?

Advanced topic

Infinite Horizon

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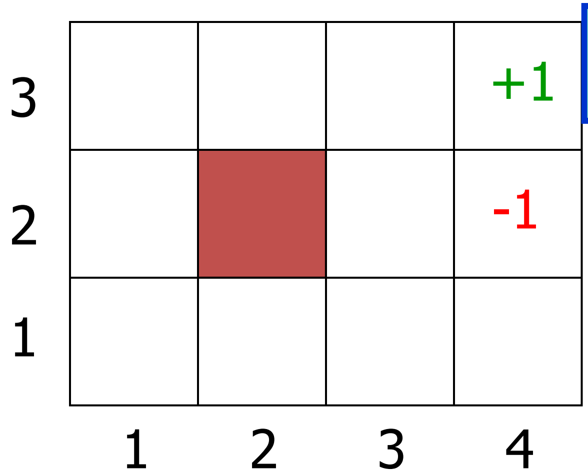

What if the robot lives forever?

One trick:
Use discounting to make an infinite horizon problem mathematically tractable

Exploration vs. Exploitation

- Problem with naïve reinforcement learning:
  - What action to take?
  - **Best apparent action, based on learning to date** } Exploitation
    - Greedy strategy
    - Often prematurely converges to a suboptimal policy!
  - **Random (or unknown) action** } Exploration
    - Will cover entire state space
    - Very expensive and slow to learn!
    - When to stop being random?
  - Balance exploration (try random actions) with exploitation (use best action so far)

More on Exploration

- Agent may sometimes choose to explore suboptimal moves in hopes of finding better outcomes
  - Only by visiting all states frequently enough can we guarantee learning the true values of all the states
- When the agent is **learning**, ideal would be to get accurate values for all states
  - Even though that may mean getting a negative outcome
- When agent is **performing**, ideal would be to get optimal outcome
- A learning agent should have an **exploration policy**

Exploration Policy

- Wacky approach (exploration): act randomly in hopes of eventually exploring entire environment
  - Choose any legal checkers move
- Greedy approach (exploitation): act to maximize utility using current estimate
  - Choose moves that have in the past led to wins
- Reasonable balance: act more wacky (exploratory) when agent has little idea of environment; more greedy when the model is close to correct
  - Suppose you know no checkers strategy?
  - What's the best way to get better?

Maintaining exploration

key ingredient of RL

deterministic/greedy policy won't explore all actions

- don't know anything about the environment at the beginning
- need to try all actions to find the optimal one

maintain exploration

- use *soft* policies instead: $\pi(s,a)>0$ (for all s,a)

ε-greedy policy

- with probability 1-ε perform the optimal/greedy action
- with probability ε perform a random action

- will keep exploring the environment
- slowly move it towards greedy policy: ε -> 0

Overview: Learning Strategies

Dynamic Programming

Q-learning

Monte Carlo approaches

Q-learning

$$Q : (s, a) \rightarrow \mathbb{R}$$

Goal: learn a function that computes a "goodness" score for taking a particular action a in state s

Q-learning

previous algorithms: on-policy algorithms

- start with a random policy, iteratively improve
- converge to optimal

Q-learning: off-policy

- use any policy to estimate Q

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha \left[r_{t+1} + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t) \right]$$

- Q directly approximates Q* (Bellman optimality equation)
- independent of the policy being followed
- only requirement: keep updating each (s,a) pair

Q-learning

previous algorithms: on-policy algorithms

- start with a random policy, iteratively improve
- converge to optimal

Learning rate, can be constant or a function

Q-learning: off-policy

- use any policy to estimate Q

R(s_t)

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha \left[r_{t+1} + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t) \right]$$

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Slide courtesy/adapted: Peter Bodík

Deep/Neural Q-learning

$$Q(s, a; \theta) \approx Q^*(s, a)$$

neural network　　　　　desired optimal solution

Deep/Neural Q-learning

$$Q(s, a; \theta) \approx Q^*(s, a)$$

neural network desired optimal solution

Approach: Form (and learn) a neural network to model our optimal Q function

Deep/Neural Q-learning

Learn weights
(parameters) θ of our
neural network

$$Q(s, a; \theta) \approx Q^*(s, a)$$

neural network

desired optimal solution

Approach: Form (and learn) a neural network to model our optimal Q function

Overview: Learning Strategies

Dynamic Programming

Q-learning

Monte Carlo approaches

Monte Carlo policy evaluation

don't need full
knowledge of
environment (just
(simulated) experience)

want to estimate $V^{\pi}(s)$

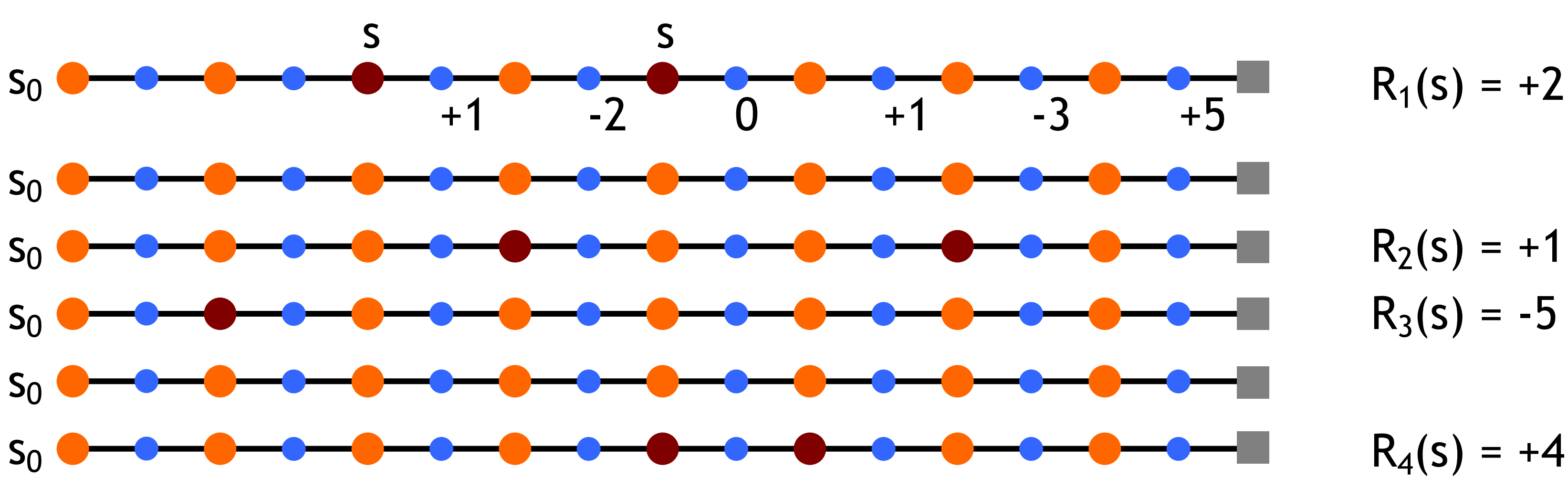
Monte Carlo policy evaluation

don't need full knowledge of environment (just **(simulated)** experience)

want to estimate $V^{\pi}(s)$

expected return **starting from s and following π**

estimate as average of **observed returns** in state s



$V^{\pi}(s) \approx (2 + 1 - 5 + 4)/4 = 0.5$

Slide courtesy/adapted: Peter Bodík

RL Summary 1:

- **Reinforcement learning systems**
  - Learn **series** of actions or decisions, rather than a single decision
  - Based on feedback given at the end of the series
- A reinforcement learner has
  - A goal
  - Carries out trial-and-error search
  - Finds the best paths toward that goal

RL Summary 2:

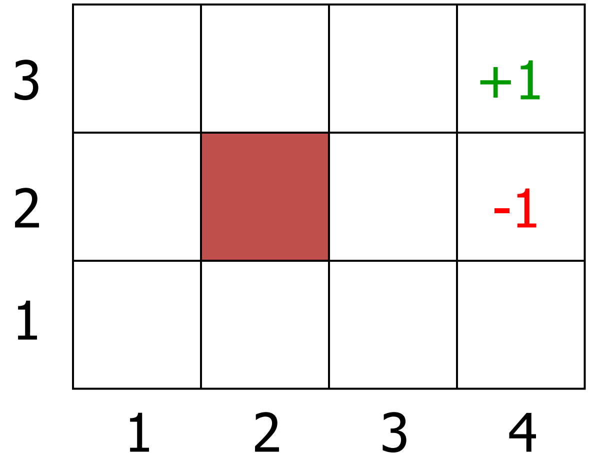
- A typical reinforcement learning system is an active agent, interacting with its environment.
- It must balance:
  - Exploration: trying different actions and sequences of actions to discover which ones work best
  - Exploitation (achievement): using sequences which have worked well so far
- Must learn **successful sequences of actions** in an uncertain environment

RL Summary 3

- Very hot area of research at the moment
- There are **many** more sophisticated RL algorithms
  - Most notably: probabilistic approaches
- Applicable to game-playing, search, finance, robot control, driving, scheduling, diagnosis, …

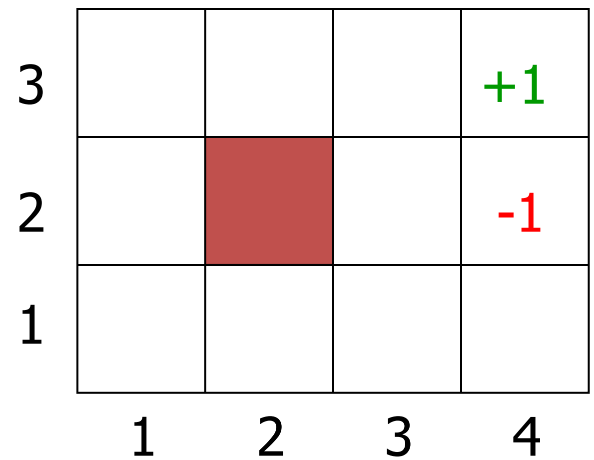
EXTRA SLIDES

Utility Function



- [4,3] provides power supply
- [4,2] is a sand area from which the robot cannot escape

Utility Function



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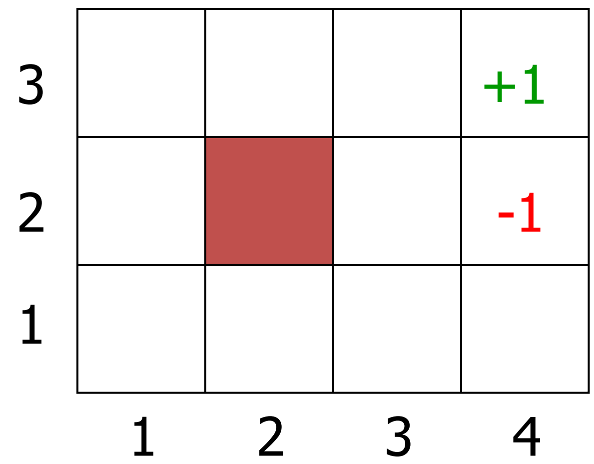
- The robot needs to recharge its batteries

Utility Function



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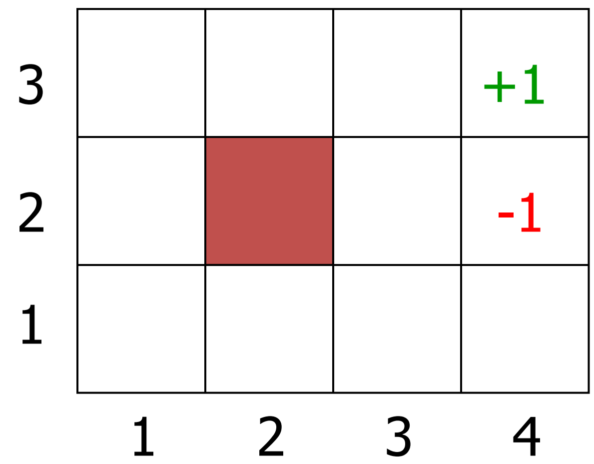
- [4,3] and [4,2] are terminal states

Utility Function



- [4,3] provides power supply
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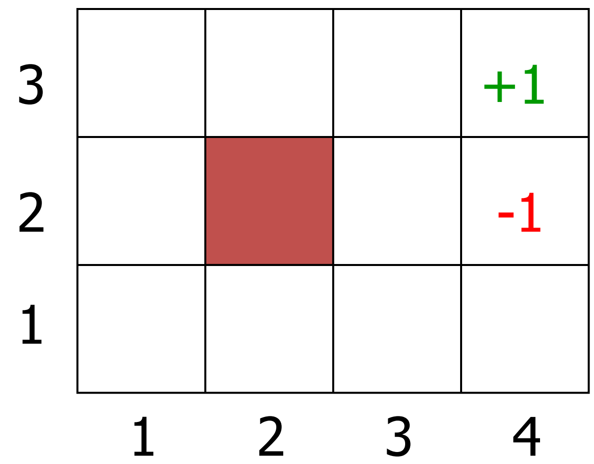
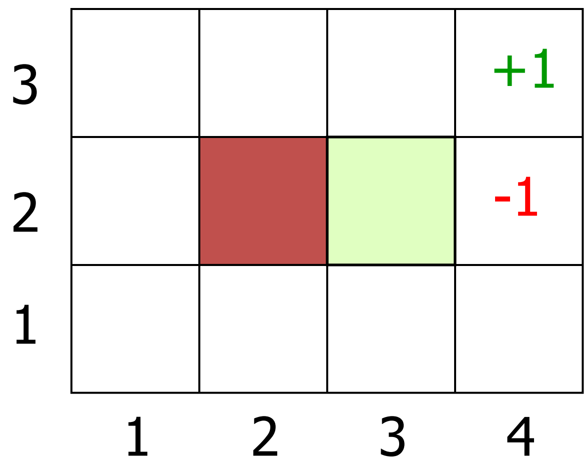
- [4,3] and [4,2] are terminal states
- Histories have utility!

Utility of a History



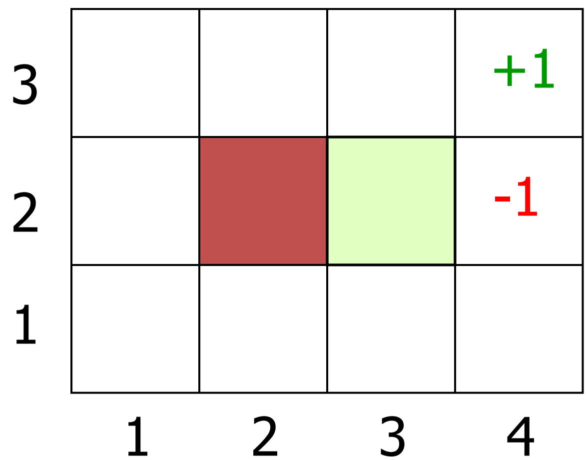
- [4,3] provides power supply
- [4,2] is a sand area from which the robot cannot escape
- The robot needs to recharge its batteries
- [4,3] or [4,2] are terminal states
- Histories have utility!
- The utility of a history is defined by the utility of the last state (+1 or –1) minus n/25, where n is the number of moves
    - Many utility functions possible, for many kinds of problems.

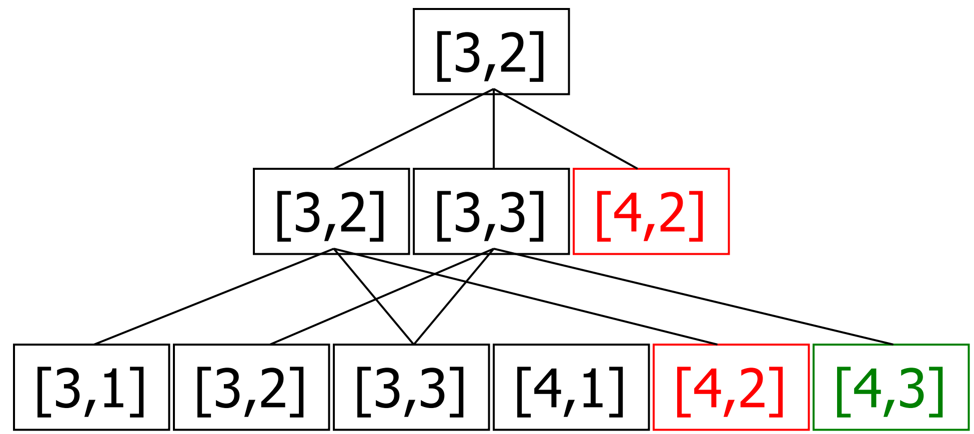
Utility of an Action Sequence



- Consider the action sequence (U,R) from [3,2]

Utility of an Action Sequence





- Consider the action sequence (U,R) from [3,2]
- A run produces one of 7 possible histories, each with some probability

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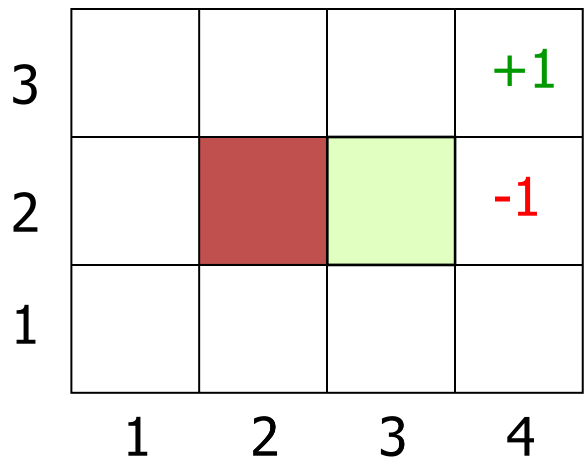
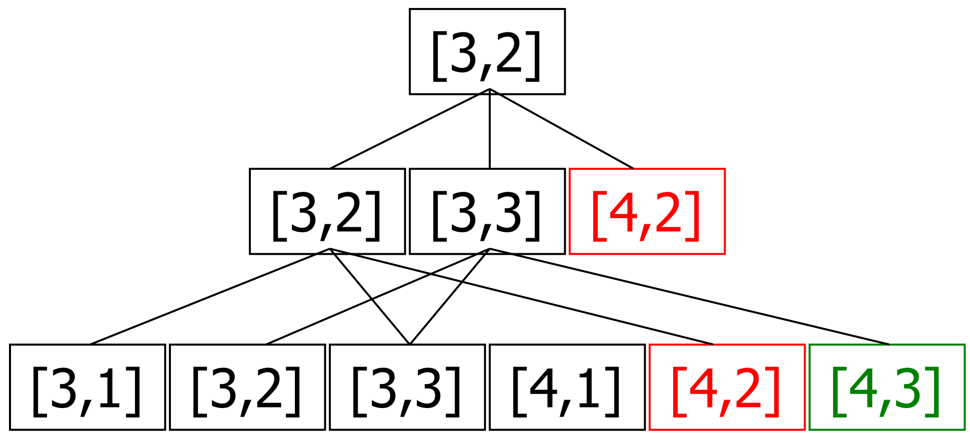
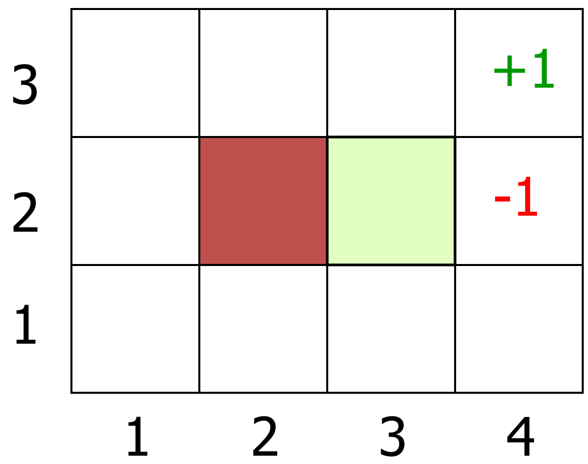


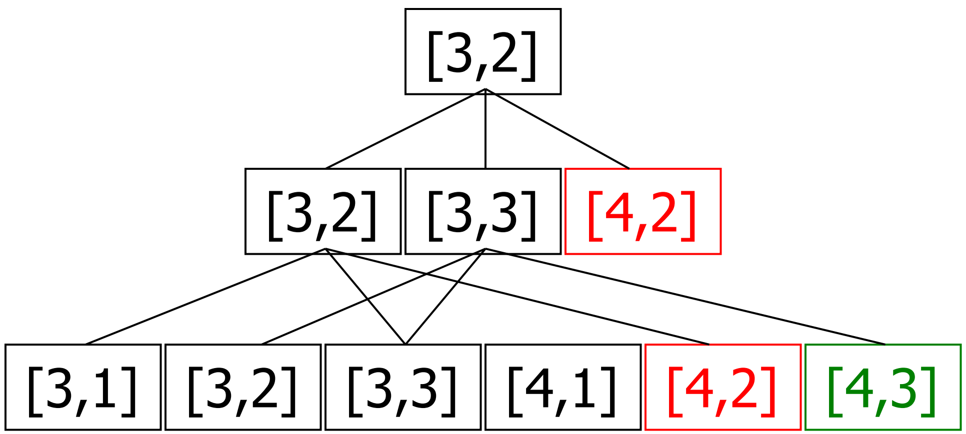


- Consider the action sequence (U,R) from [3,2]
- A run produces one of 7 possible histories, each with some probability
- The utility of the sequence is the expected utility of the histories:

$$\mathcal{U} = \Sigma_h \mathcal{U}_h \mathbf{P}(h)$$

Optimal Action Sequence





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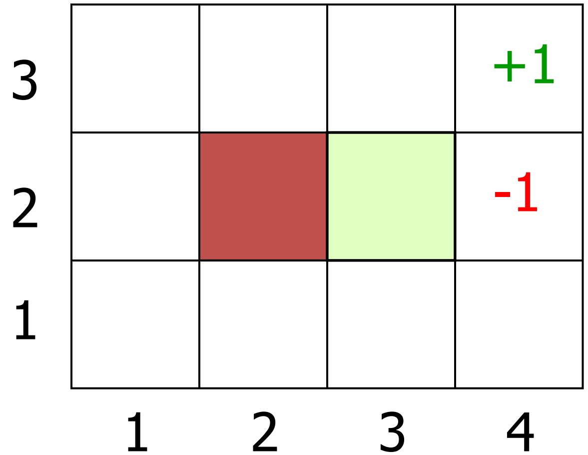

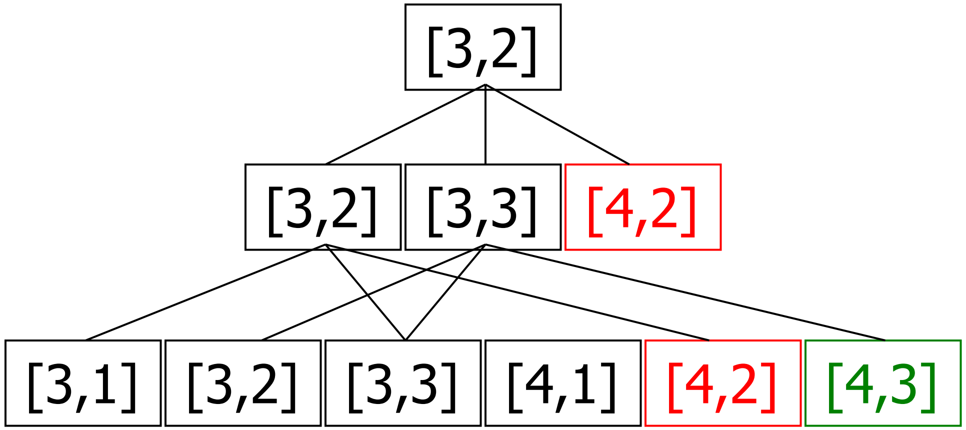

- Consider the action sequence (U,R) from [3,2]
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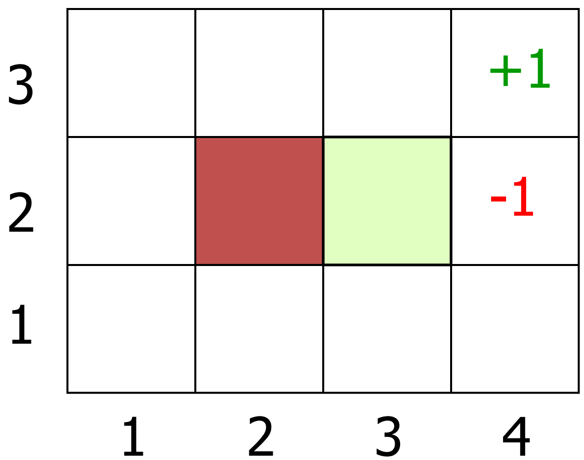
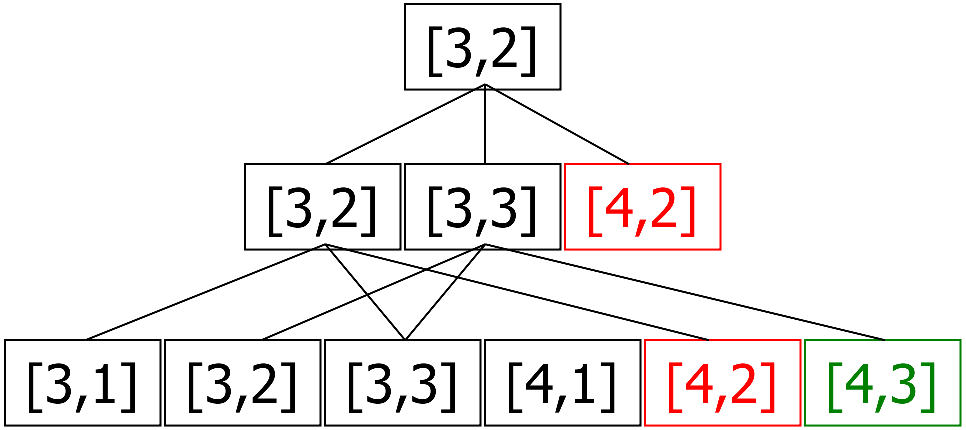
- **But is the optimal action sequence what we want to compute?**

Optimal Action Sequence





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only if the sequence is executed blindly!