Machine Learning

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UMBC CMSC 471

Today:

- Naïve Bayes
  - discrete-valued X_i's
  - Document classification
- Gaussian Naïve Bayes
  - real-valued X_i's
  - Brain image classification

Recently:

- Bayes classifiers to learn P(Y|X)
- MLE and MAP estimates for parameters of P
- Conditional independence
- Naïve Bayes → make Bayesian learning practical

Next:

- Text classification
- Naïve Bayes and continuous variables X_i:
  - Gaussian Naïve Bayes classifier
- Learn P(Y|X) directly
  - Logistic regression, Regularization, Gradient ascent
- Naïve Bayes or Logistic Regression?
  - Generative vs. Discriminative classifiers

Estimating Parameters

- Maximum Likelihood Estimate (MLE): choose θ that maximizes probability of observed data $\mathcal{D}$

$$\widehat{\theta} = \arg\max_{\theta} \; P(\mathcal{D} \mid \theta)$$

- Maximum a Posteriori (MAP) estimate: choose θ that is most probable given prior probability and the data

$$\widehat{\theta} = \arg\max_{\theta} \; P(\theta \mid \mathcal{D})$$

$$= \arg\max_{\theta} \; = \frac{P(\mathcal{D} \mid \theta)P(\theta)}{P(\mathcal{D})}$$

Estimating Parameters: Y, X_i discrete-valued

Maximum likelihood estimates:

$$\hat{\pi}_k = \hat{P}(Y = y_k) = \frac{\#D\{Y = y_k\}}{|D|}$$

$$\hat{\theta}_{ijk} = \hat{P}(X_i = x_j | Y = y_k) = \frac{\#D\{X_i = x_j \wedge Y = y_k\}}{\#D\{Y = y_k\}}$$

MAP estimates (Beta, Dirichlet priors):

$$\hat{\pi}_k = \hat{P}(Y = y_k) = \frac{\#D\{Y = y_k\} + (\beta_k - 1)}{|D| + \sum_m (\beta_m - 1)}$$

$$\hat{\theta}_{ijk} = \hat{P}(X_i = x_j | Y = y_k) = \frac{\#D\{X_i = x_j \wedge Y = y_k\} + (\beta_k - 1)}{\#D\{Y = y_k\} + \sum_m (\beta_m - 1)}$$

Only difference:
"imaginary" examples

Naïve Bayes: classifying text documents

- Classify which emails are spam?
- Classify which emails promise an attachment?

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>
> ********************************
>
> Randal E. Bryant
> Dean and University Professor

How shall we represent text documents for Naïve Bayes?

Learning to classify documents: $P(Y|X)$

- Y discrete valued.
  - e.g., Spam or not
- $X = \langle X_1, X_2, \ldots X_n \rangle$ = document

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- X_i is a random variable describing…

Answer 1: X_i is boolean, 1 if word i is in document, else 0

e.g., $X_{pleased} = 1$

Issues?

Learning to classify documents: P(Y|X)

- Y discrete valued.
  - e.g., Spam or not
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- X_i is a random variable describing…

Answer 2:

- X_i represents the i^{th} *word position* in document
- X_1 = "I", X_2 = "am", X_3 = "pleased"
- and, let's assume the X_i are iid (indep, identically distributed)

$$P(X_i|Y) = P(X_j|Y) \quad (\forall i, j)$$

Learning to classify document: P(Y|X) the "Bag of Words" model

- Y discrete valued. e.g., Spam or not
- $X = <X_1, X_2, \ldots X_n>$ = document

- X_i are iid random variables. Each represents the word at its position i in the document
- Generating a document according to this distribution = rolling a 50,000 sided die, once for each word position in the document

- The observed counts for each word follow a ??? distribution

Multinomial Distribution

Eg. 2 Dice roll problem (6 outcomes instead of 2)

Likelihood is ~ Multinomial($\theta = \{\theta_1, \theta_2, \ldots, \theta_k\}$)

$$P(\mathcal{D} \mid \theta) = \theta_1^{\alpha_1} \theta_2^{\alpha_2} \ldots \theta_k^{\alpha_k}$$

If prior is Dirichlet distribution,

$$P(\theta) = \frac{\prod_{i=1}^{k} \theta_i^{\beta_i - 1}}{B(\beta_1, \ldots, \beta_k)} \sim \text{Dirichlet}(\beta_1, \ldots, \beta_k)$$

Then posterior is Dirichlet distribution

$$P(\theta|D) \sim \text{Dirichlet}(\beta_1 + \alpha_1, \ldots, \beta_k + \alpha_k)$$

$$\hat{\theta}_i^{MAP} = \hat{P}(X = i) = \frac{\alpha_i + \beta_i - 1}{\sum_{j=1}^{k}(\alpha_j + \beta_j - 1)}$$

Multinomial Distribution

k sides

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$$P(\mathcal{D} \mid \theta) = \theta_1^{\alpha_1} \theta_2^{\alpha_2} \ldots \theta_k^{\alpha_k}$$

$\theta_i = P(X = i)$

If prior is Dirichlet distribution,

$$P(\theta) = \frac{\prod_{i=1}^{k} \theta_i^{\beta_i - 1}}{B(\beta_1, \ldots, \beta_k)} \sim \text{Dirichlet}(\beta_1, \ldots, \beta_k)$$

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$$P(\theta|D) \sim \text{Dirichlet}(\beta_1 + \alpha_1, \ldots, \beta_k + \alpha_k)$$

$$\hat{\theta}_i^{MAP} = \hat{P}(X = i) = \frac{\alpha_i + \beta_i - 1}{\sum_{j=1}^{k} (\alpha_j + \beta_j - 1)}$$

Dirichlet is a generalization of Beta Distribution

Multinomial Bag of Words

counts / α_i's

→

aardvark	0
about	2
all	2
Africa	1
apple	0
anxious	0
...	
gas	1
...	
oil	1
…	
Zaire	0

MAP estimates for bag of words

Map estimate for multinomial

$$\hat{\theta}_i^{MAP} = \hat{P}(X = i) = \frac{\alpha_i + \beta_i - 1}{\sum_{j=1}^{k}(\alpha_j + \beta_j - 1)}$$

$$\hat{\theta}_{aardvark}^{MAP} = P(X = \text{aardvark}) = \frac{\#\text{ observed 'aardvark' } + \#\text{ hallucinated 'aardvark'}}{\#\text{ observed words } + \#\text{ hallucinated words}}$$

What β's should we choose?

MAP estimates for bag of words

Map estimate for multinomial

$$\hat{\theta}_i^{MAP} = \hat{P}(X = i) = \frac{\alpha_i + \beta_i - 1}{\sum_{j=1}^{k}(\alpha_j + \beta_j - 1)}$$

$$\hat{\theta}_{aardvark}^{MAP} = P(X = \text{aardvark}) = \frac{\#\ \text{observed 'aardvark'} \ + \#\ \text{hallucinated 'aardvark'}}{\#\ \text{observed words} \ + \#\ \text{hallucinated words}}$$

What β's should we choose?

- Large document, how many times each word occur
- Uniform distribution

Naïve Bayes Algorithm – discrete X_i

- Train Naïve Bayes (examples)

  for each value y_k

  estimate $\pi_k \equiv P(Y = y_k)$

  for each value x_{ij} of each attribute X_i

  estimate $\theta_{ijk} \equiv P(X_i = x_{ij} | Y = y_k)$

  prob that word x_{ij} appears in position i, given $Y=y_k$

- Classify (X^{new})

$$Y^{new} \leftarrow \arg\max_{y_k} \; P(Y = y_k) \prod_i P(X_i^{new} | Y = y_k)$$

$$Y^{new} \leftarrow \arg\max_{y_k} \; \pi_k \prod_i \theta_{ijk}$$

* Additional assumption: word probabilities are position independent

$$\theta_{ijk} = \theta_{mjk} \;\; \text{for} \;\; i \neq m$$

Naïve Bayes Algorithm – discrete X_i

- Train Naïve Bayes (examples)

  for each value y_k

  estimate $\pi_k \equiv P(Y = y_k)$

  spam : k = 1
  ¬spam : k = 0

  for each value x_{ij} of each attribute X_i

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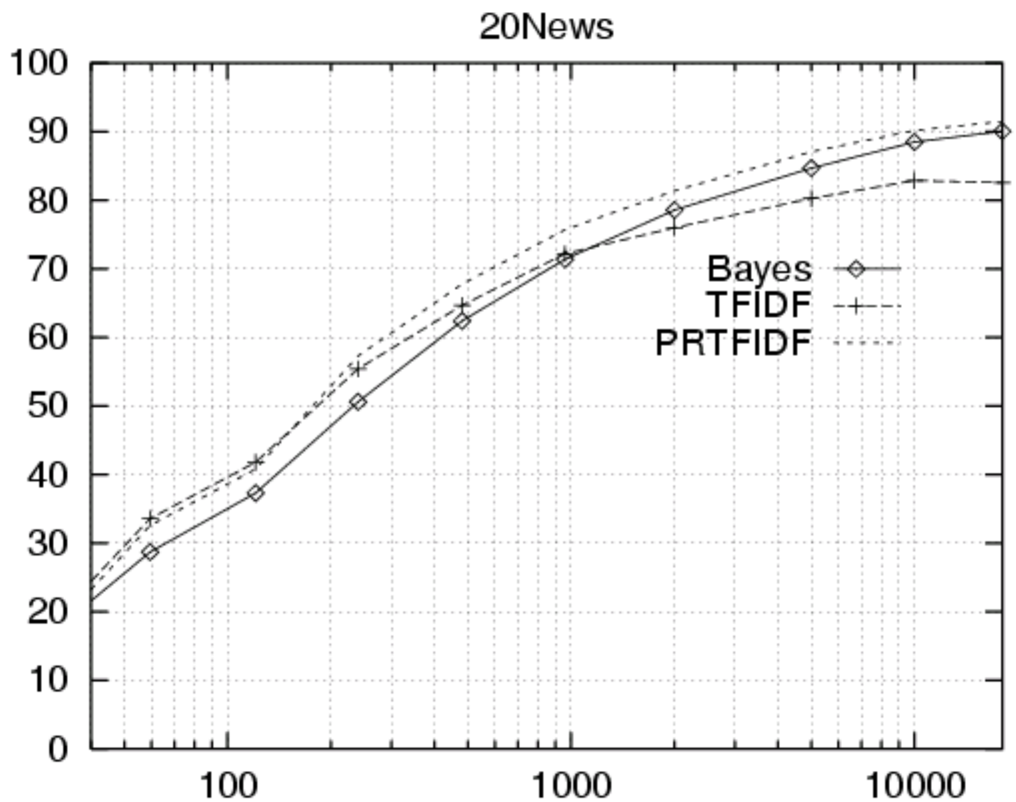
Twenty NewsGroups

Given 1000 training documents from each group
Learn to classify new documents according to which newsgroup it came from

comp.graphics	misc.forsale
comp.os.ms-windows.misc	rec.autos
comp.sys.ibm.pc.hardware	rec.motorcycles
comp.sys.mac.hardware	rec.sport.baseball
comp.windows.x	rec.sport.hockey
alt.atheism	sci.space
soc.religion.christian	sci.crypt
talk.religion.misc	sci.electronics
talk.politics.mideast	sci.med
talk.politics.misc	
talk.politics.guns	

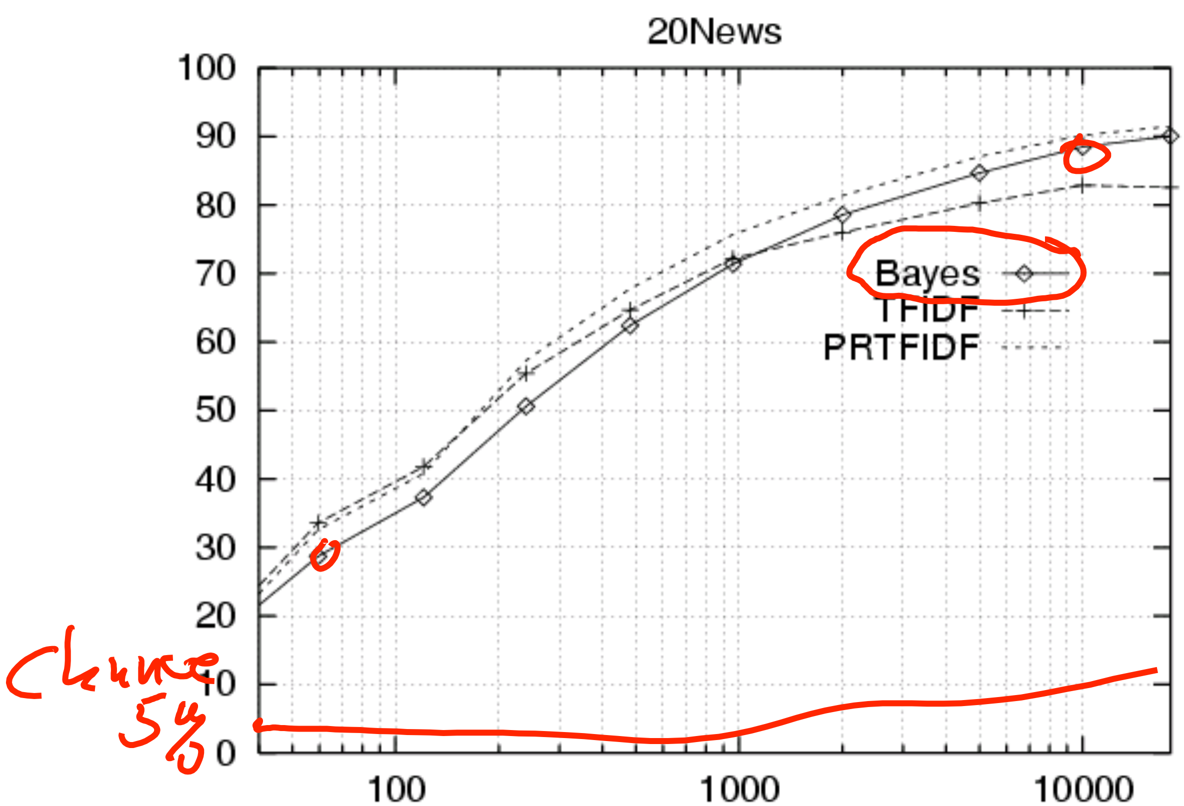
Naive Bayes: 89% classification accuracy

Learning Curve for 20 Newsgroups



Accuracy vs. Training set size (1/3 withheld for test)

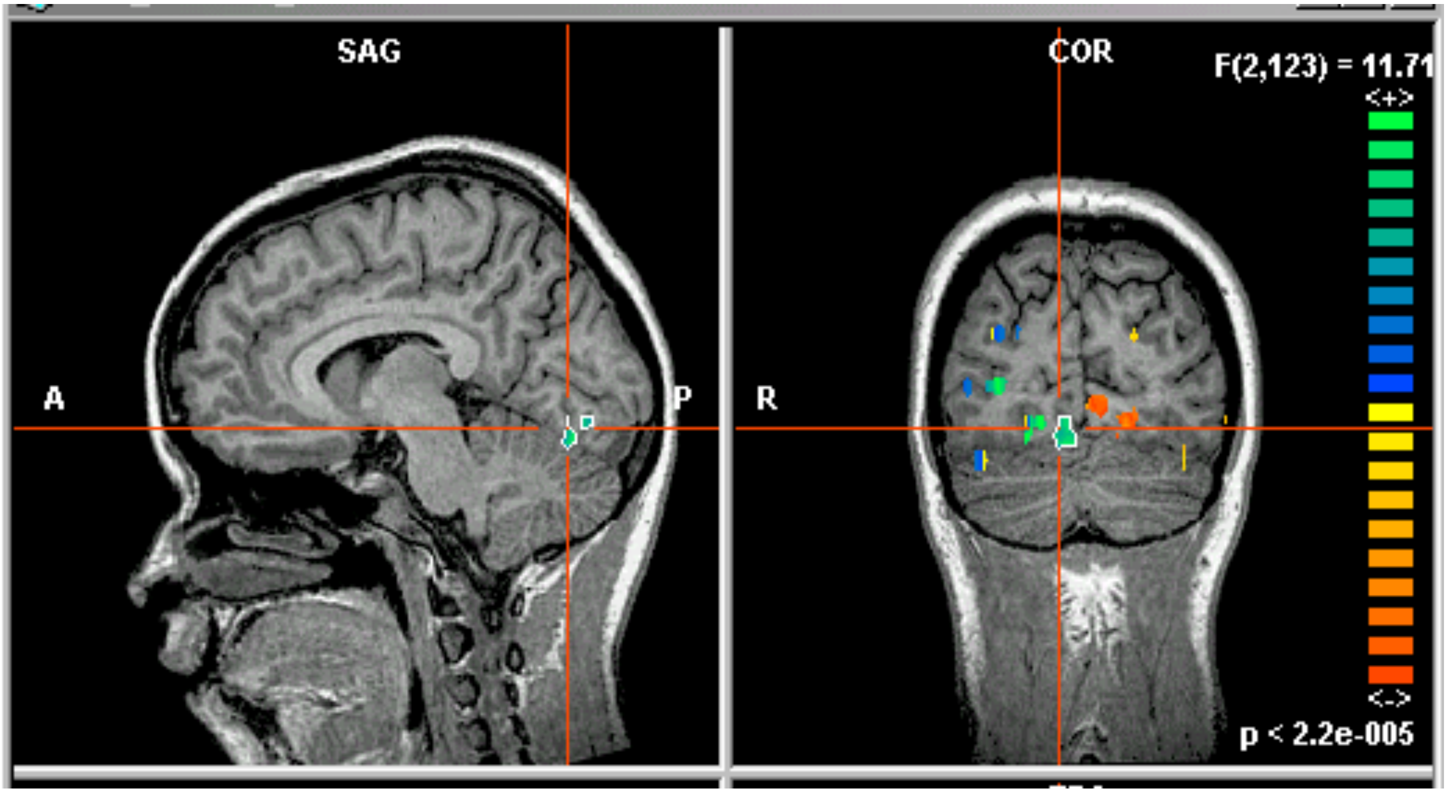
Learning Curve for 20 Newsgroups



Accuracy vs. Training set size (1/3 withheld for test)

What if we have continuous X_i ?

Eg., image classification: X_i is real-valued i^th pixel



What if we have continuous X_i ?

Eg., image classification: X_i is real-valued i[th] pixel

Naïve Bayes requires $P(X_i \mid Y{=}y_k)$, but X_i is real (continuous)

$$P(Y = y_k | X_1 \ldots X_n) = \frac{P(Y = y_k) \prod_i P(X_i | Y = y_k)}{\sum_j P(Y = y_j) \prod_i P(X_i | Y = y_j)}$$

Common approach: assume $P(X_i \mid Y{=}y_k)$ follows a Normal (Gaussian) distribution

What if we have continuous X_i ?

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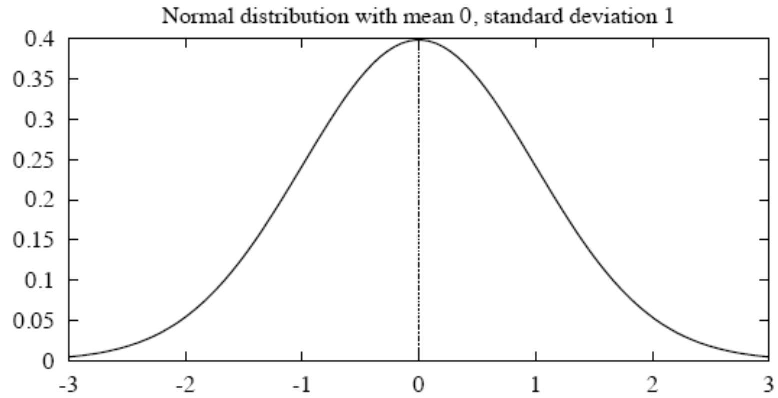
Common approach: assume $P(X_i \mid Y=y_k)$ follows a Normal (Gaussian) distribution

Gaussian Distribution
(also called "Normal")

p(x) is a *probability density function*, whose integral (not sum) is 1



$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

The probability that X will fall into the interval (a, b) is given by

$$\int_a^b p(x)dx$$

- Expected, or mean value of X, $E[X]$, is

$$E[X] = \mu$$

- Variance of X is

$$Var(X) = \sigma^2$$

- Standard deviation of X, σ_X, is

$$\sigma_X = \sigma$$

What if we have continuous X_i ?

Gaussian Naïve Bayes (GNB): assume

$$p(X_i = x|Y = y_k) = \frac{1}{\sqrt{2\pi\sigma_{ik}^2}} \; e^{-\frac{1}{2}(\frac{x-\mu_{ik}}{\sigma_{ik}})^2}$$

Sometimes assume variance

- is independent of Y (i.e., σ_i),
- or independent of X_i (i.e., σ_k)
- or both (i.e., σ)

Gaussian Naïve Bayes Algorithm – continuous X_i (but still discrete Y)

- Train Naïve Bayes (examples)

  for each value y_k

    estimate* $\pi_k \equiv P(Y = y_k)$

    for each attribute X_i estimate $P(X_i|Y = y_k)$

    - class conditional mean μ_{ik}, variance σ_{ik}

- Classify (X^{new})

$$Y^{new} \leftarrow \arg\max_{y_k} \; P(Y = y_k) \prod_i P(X_i^{new}|Y = y_k)$$

$$Y^{new} \leftarrow \arg\max_{y_k} \; \pi_k \prod_i \mathcal{N}(X_i^{new}; \mu_{ik}, \sigma_{ik})$$

* probabilities must sum to 1, so need estimate only n-1 parameters...

Estimating Parameters: Y discrete, X_i continuous

Maximum likelihood estimates:

jth training example

$$\widehat{\mu}_{ik} = \frac{1}{\sum_j \delta(Y^j = y_k)} \sum_j X_i^j \delta(Y^j = y_k)$$

ith feature

kth class

δ()=1 if (Y^j=y_k) else 0

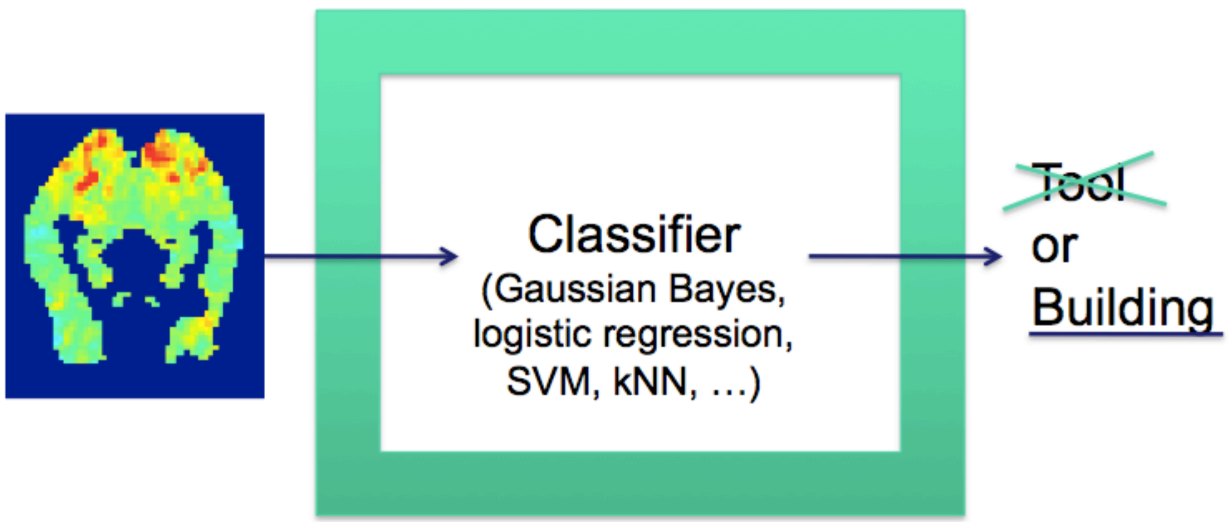
$$\widehat{\sigma}_{ik}^2 = \frac{1}{\sum_j \delta(Y^j = y_k)} \sum_j (X_i^j - \widehat{\mu}_{ik})^2 \delta(Y^j = y_k)$$

How many parameters must we estimate for Gaussian Naïve Bayes if Y has k possible values, X=<X1, … Xn>?

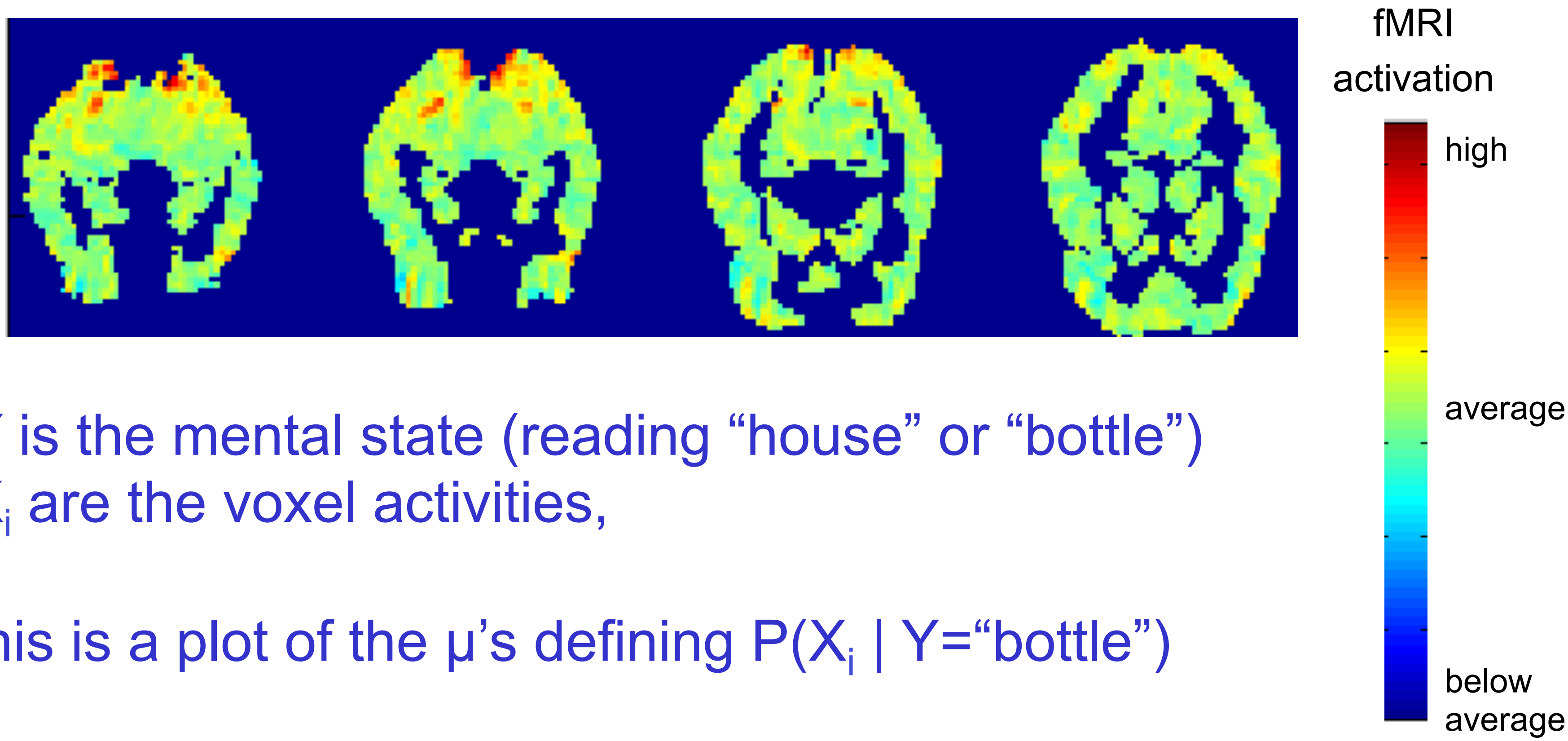
$$p(X_i = x | Y = y_k) = \frac{1}{\sqrt{2\pi\sigma_{ik}^2}} \; e^{-\frac{1}{2}(\frac{x-\mu_{ik}}{\sigma_{ik}})^2}$$

GNB Example: Classify a person's cognitive state, based on brain image

- reading a sentence or viewing a picture?
- reading the word describing a "Tool" or "Building"?
- answering the question, or getting confused?



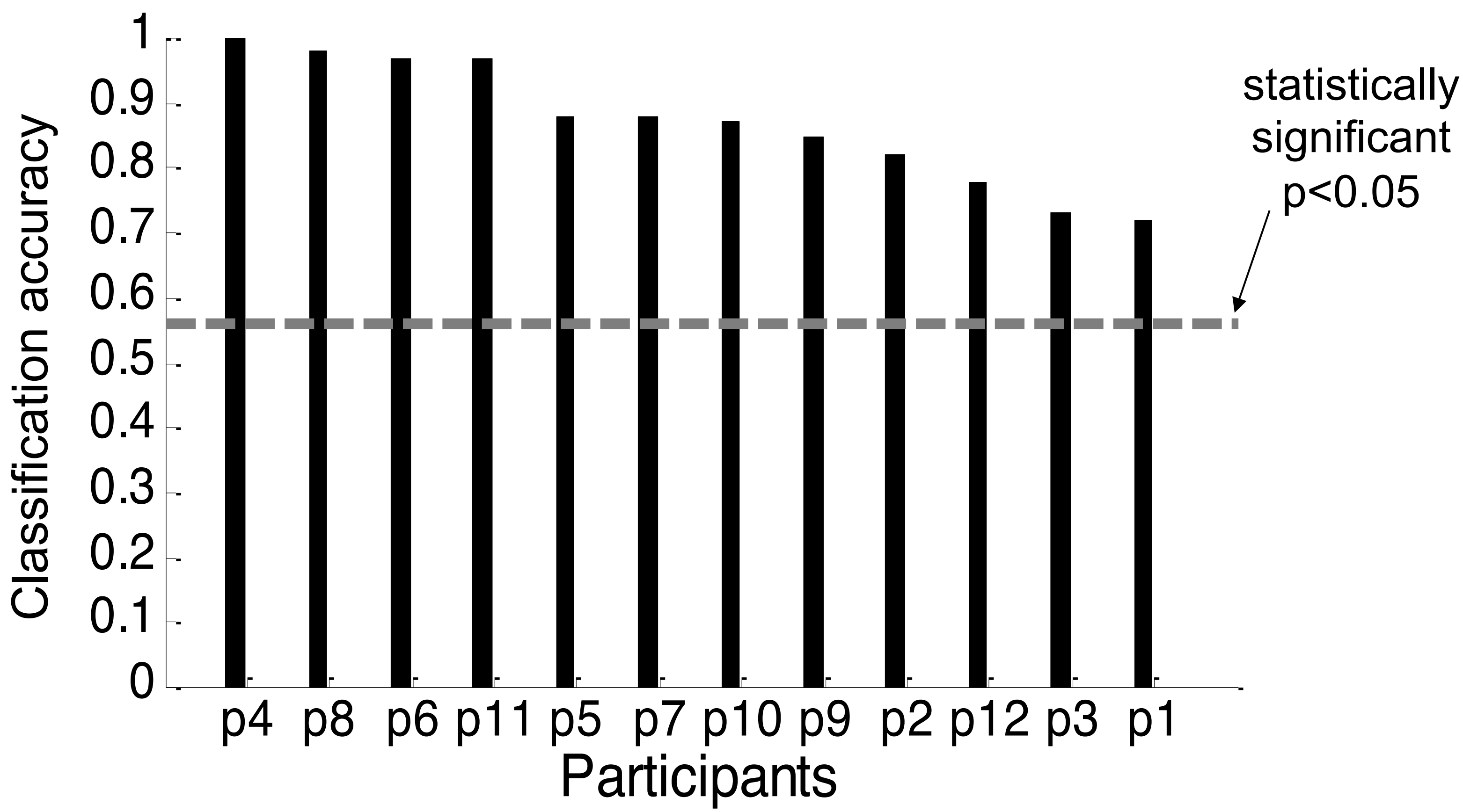
Mean activations over all training examples for Y="bottle"



Y is the mental state (reading "house" or "bottle")
X_i are the voxel activities,

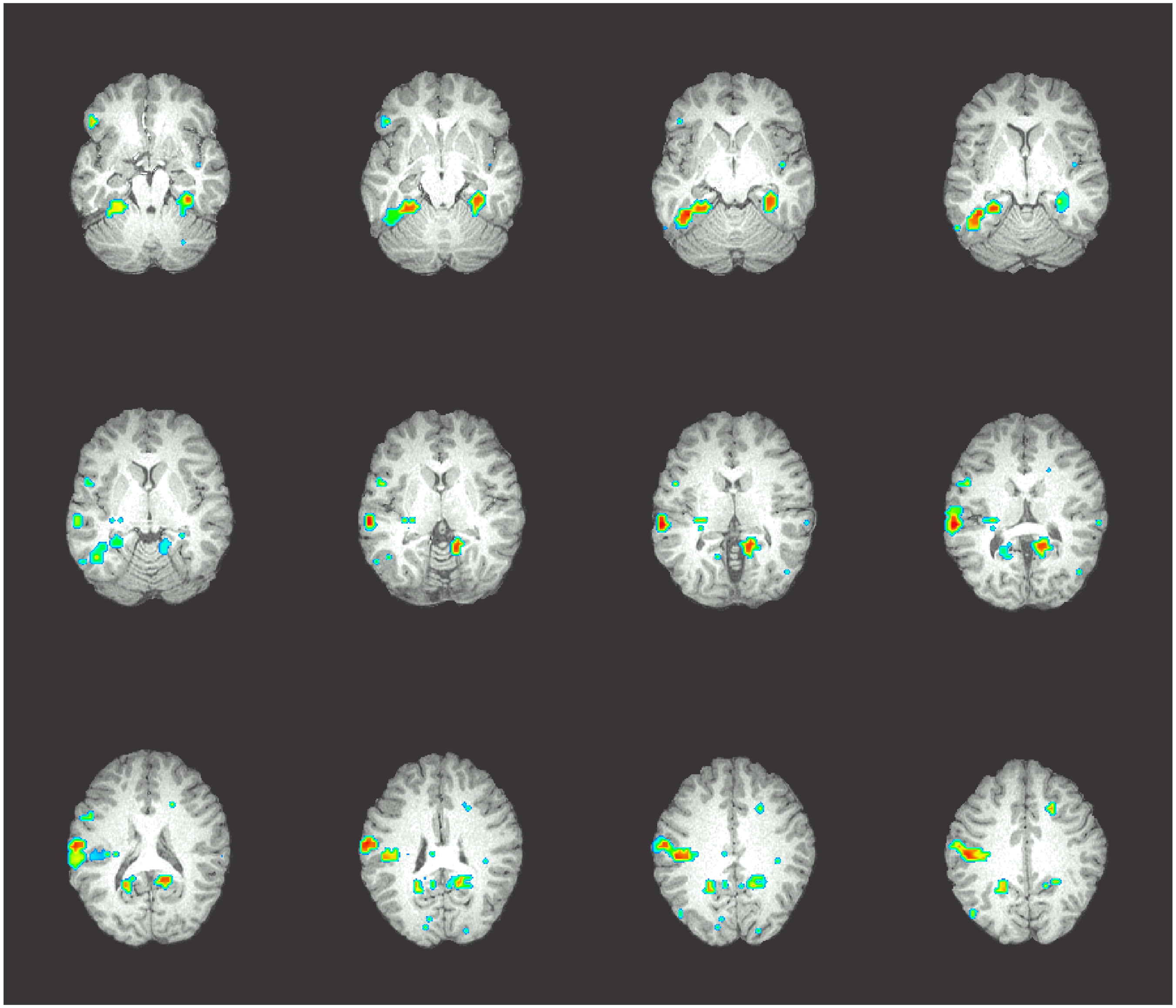
this is a plot of the μ's defining P(X_i | Y="bottle")

Classification task: is person viewing a “tool” or “building”?



Where is information encoded in the brain?

Accuracies of cubical 27-voxel classifiers centered at each significant voxel [0.7-0.8]



Naïve Bayes: What you should know

- Designing classifiers based on Bayes rule
- Conditional independence
  - What it is
  - Why it's important
- Naïve Bayes assumption and its consequences
  - Which (and how many) parameters must be estimated under different generative models (different forms for P(X|Y))
    - and why this matters
- How to train Naïve Bayes classifiers
  - MLE and MAP estimates
  - with discrete and/or continuous inputs X_i

Questions to think about:

- Can you use Naïve Bayes for a combination of discrete and real-valued X_i?
- How can we easily model just 2 of n attributes as dependent?
- What does the decision surface of a Naïve Bayes classifier look like?
- How would you select a subset of X_i's?