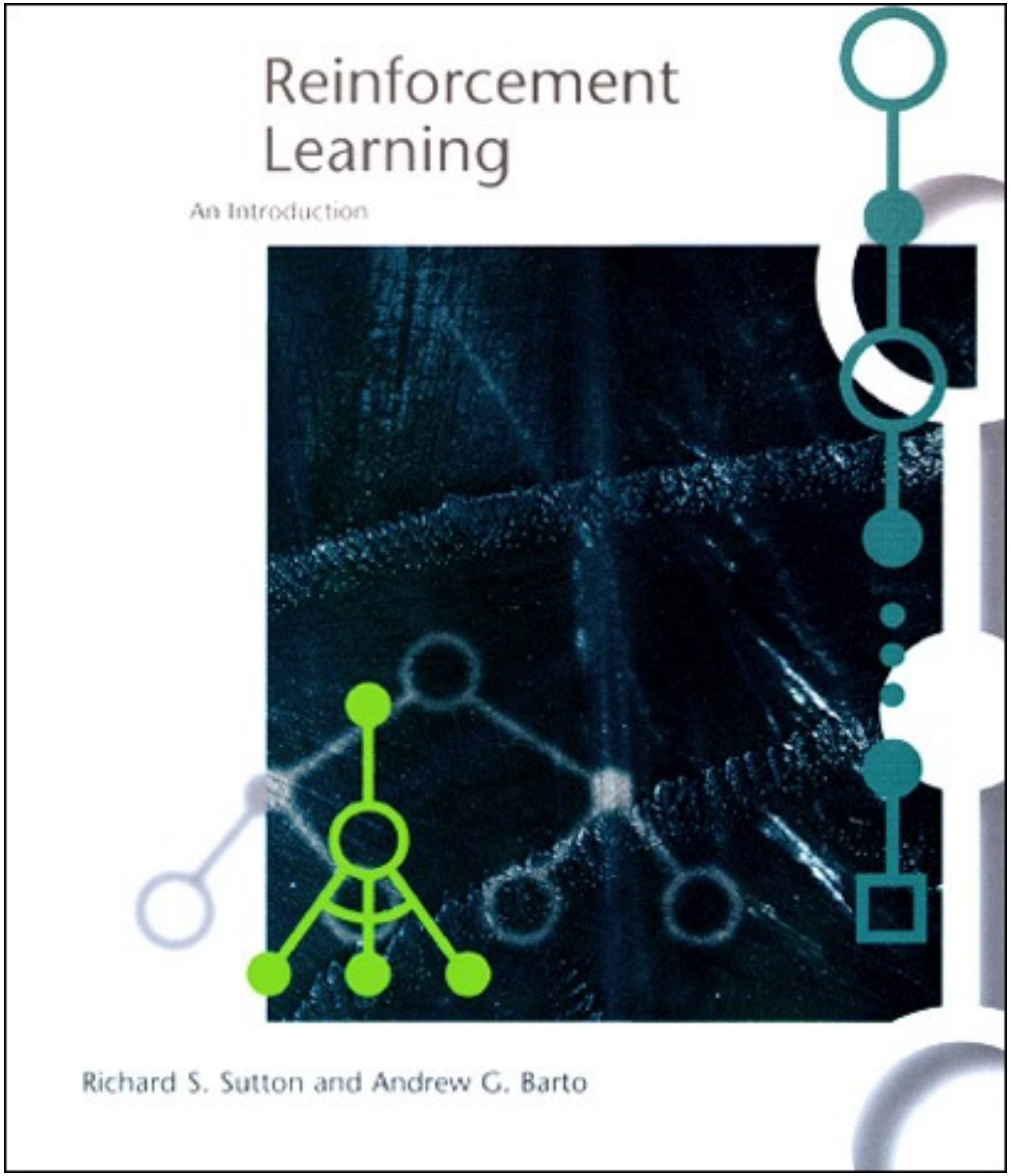
CMSC 478: Reinforcement Learning

Some slides courtesy Cynthia Matuszek and Frank Farrero, with some material from Marie desJardin, Lise Getoor, Jean-Claude Latombe, and Daphne Koller

There's an entire book!

http://incompleteideas.net/book/the-book-2nd.html



The Big Idea

- "Planning": Find a sequence of steps to accomplish a goal.
  - Given start state, transition model, goal functions…
- This is a kind of **sequential decision making**.
  - Transitions are deterministic.
- What if they are stochastic (probabilistic)?
  - One time in ten, you drop your sock
- Probabilistic Planning: Make a plan that accounts for probability by **carrying it through the plan**.

Review: Formalizing Agents

- Given:
  – A state space S
  – A set of actions $a_1, \ldots, a_k$ including their results
  – Reward value at the end of each trial (series of action) (may be positive or negative)
- Output:
  – A **mapping from states to actions**

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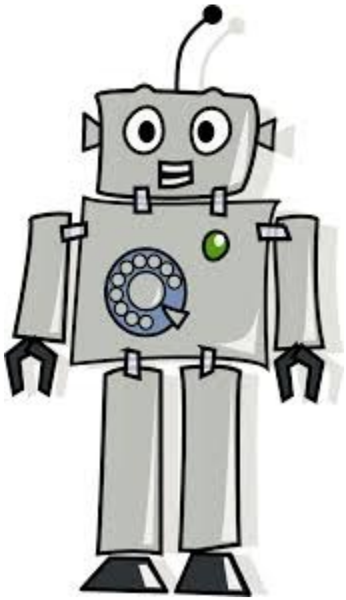
- Output:
  - A **mapping from states to actions**
  - Which is a **policy**, π

Reinforcement Learning

- We often have an agent which has a **task** to perform
  - It takes some actions in the world
  - At some later point, gets feedback on how well it did
  - The agent performs the same task repeatedly
- This problem is called **reinforcement learning**:
  - The agent gets positive reinforcement for tasks done well
  - And gets negative reinforcement for tasks done poorly
  - Must somehow figure out which actions to take next time

Reinforcement Learning



agent

environment

Robot image: openclipart.org

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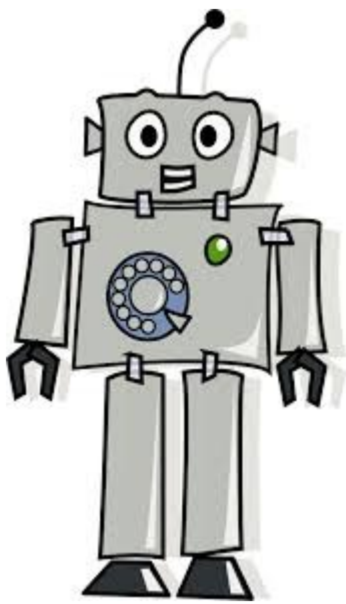
Reinforcement Learning

take action



agent

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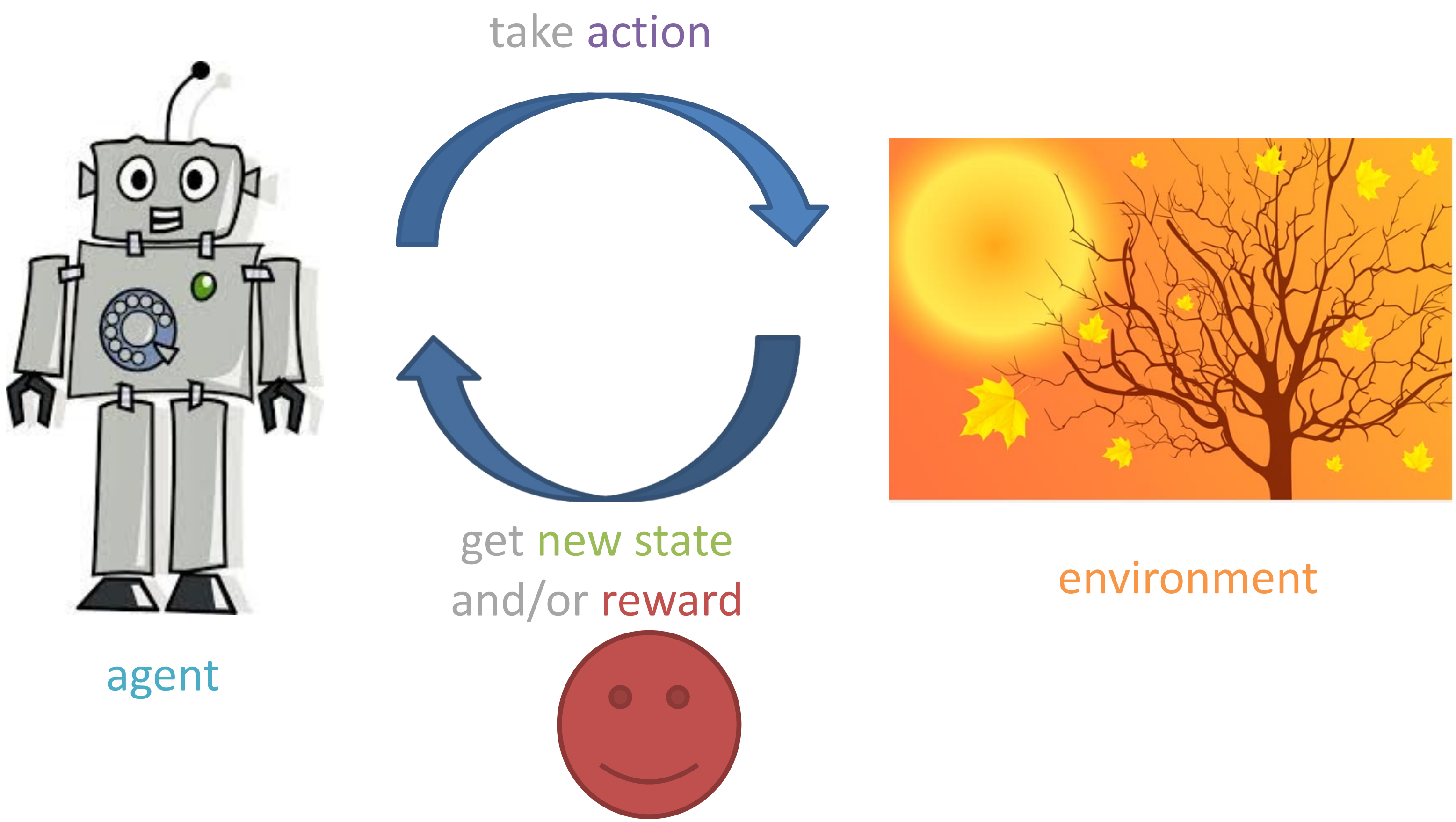
Reinforcement Learning



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Reinforcement Learning

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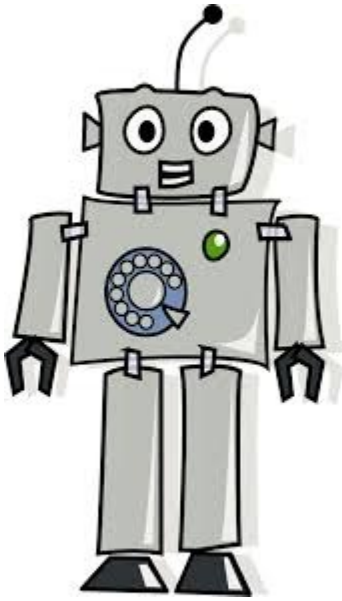

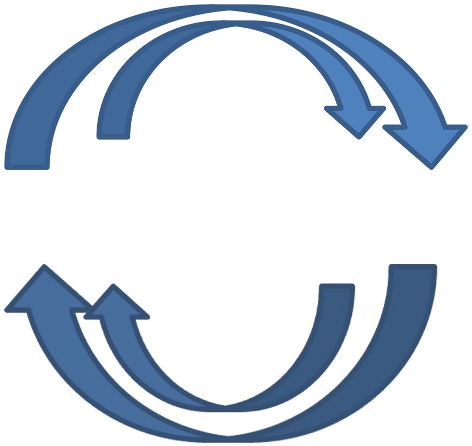

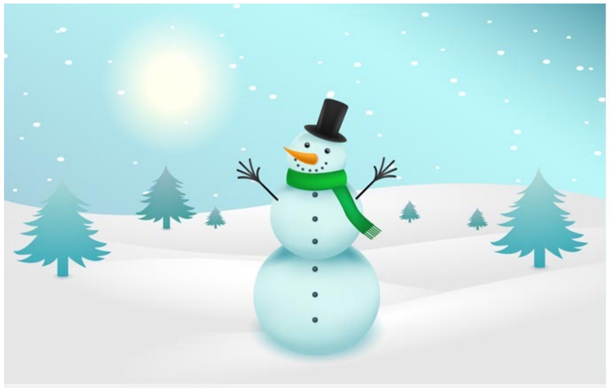

get new state
and/or reward

environment

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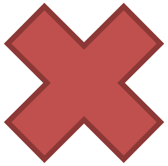

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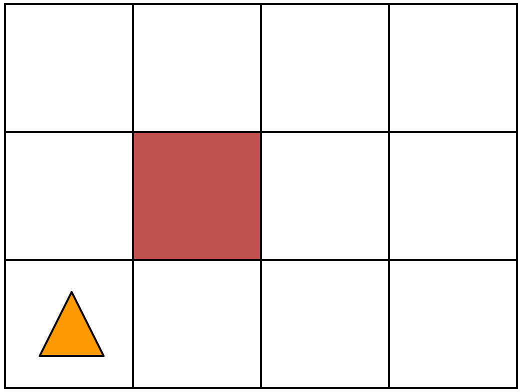
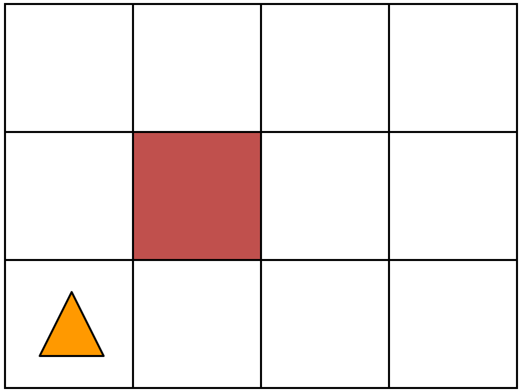
Simple Robot Navigation Problem



- In each state, the possible actions are U, D, R, and L

Probabilistic Transition Model



- In each state, the possible actions are U, D, R, and L
- The effect of U is as follows (transition model):
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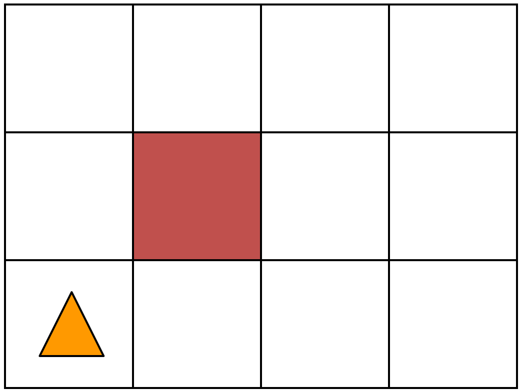

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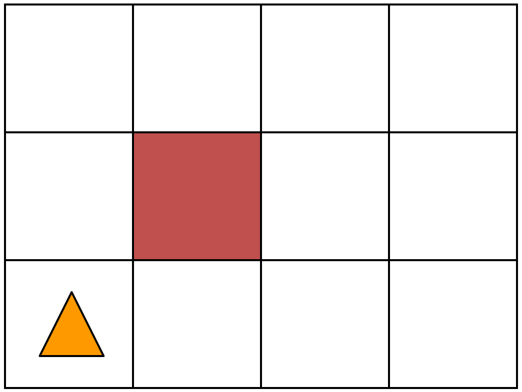

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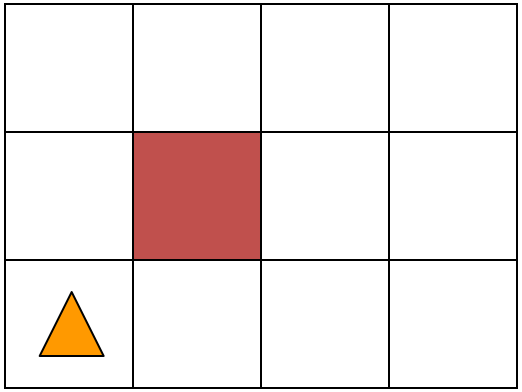
Probabilistic Transition Model



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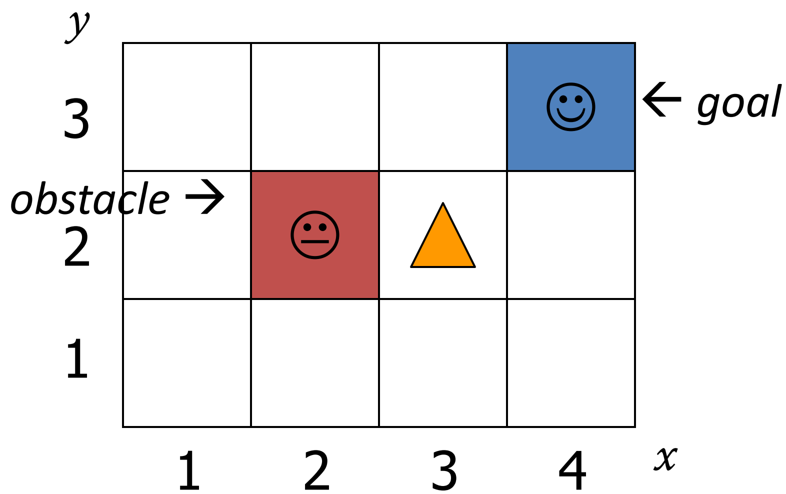
- D, R, and L have similar probabilistic effects

Markov Property

The transition properties depend only on the current state, not on the previous history (how that state was reached)

Markov assumption generally: current state only ever depends on previous state (or finite set of previous states).

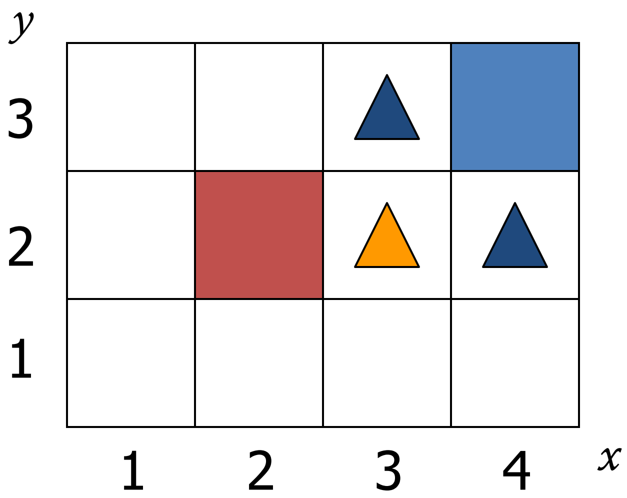
Sequence of Actions

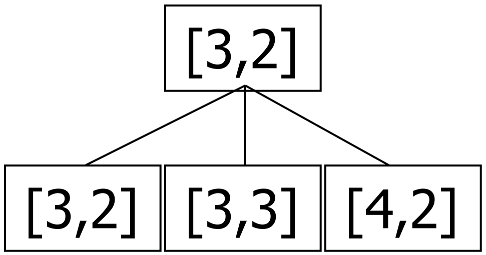


[3,2] ← *start state*

- Planned sequence of actions: (U, R)

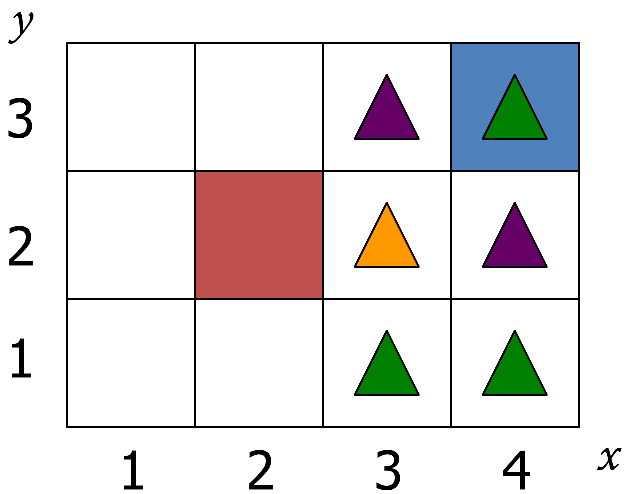
Sequence of Actions

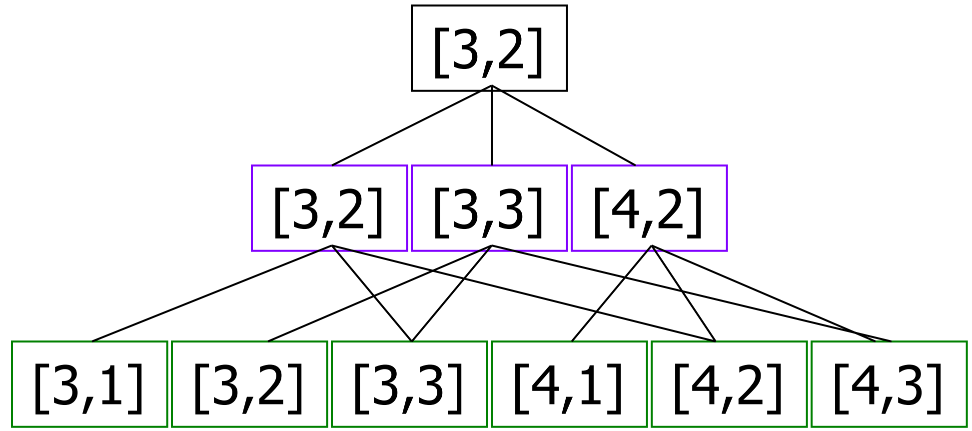




- Planned sequence of actions: (U, R)
- U is executed

Histories

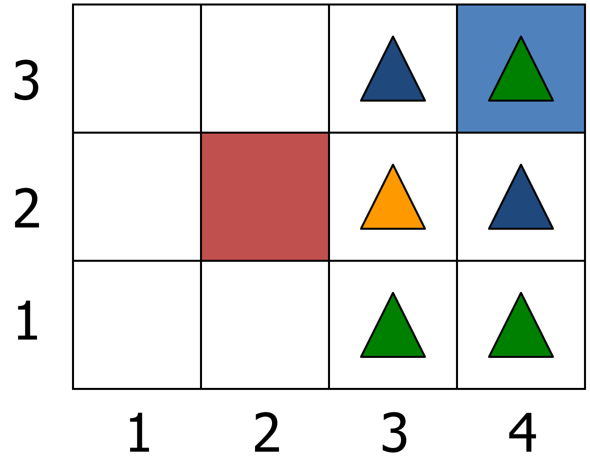




- Planned sequence of actions: (U, R)
- U has been executed
- R is executed

- 9 possible sequences of states – called histories
- 6 possible final states for the robot!

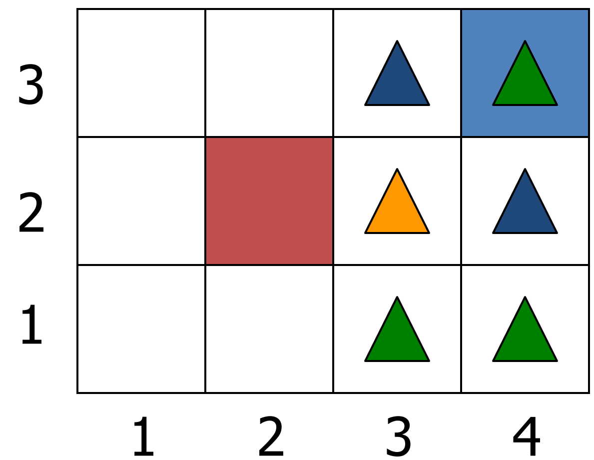
Probability of Reaching the Goal



- $\mathbf{P}([4,3] \mid (U,R).[3,2]) =$

$$\mathbf{P}([4,3] \mid R.[3,3]) \times \mathbf{P}([3,3] \mid U.[3,2])$$
$$+\ \mathbf{P}([4,3] \mid R.[4,2]) \times \mathbf{P}([4,2] \mid U.[3,2])$$

Probability of Reaching the Goal



- **P**([4,3] | (U,R).[3,2]) =

  P([4,3] | R.[3,3]) x **P**([3,3] | U.[3,2])

  \+ **P**([4,3] | R.[4,2]) x **P**([4,2] | U.[3,2])

  - **P**([3,3] | U.[3,2]) = 0.8
  - **P**([4,2] | U.[3,2]) = 0.1

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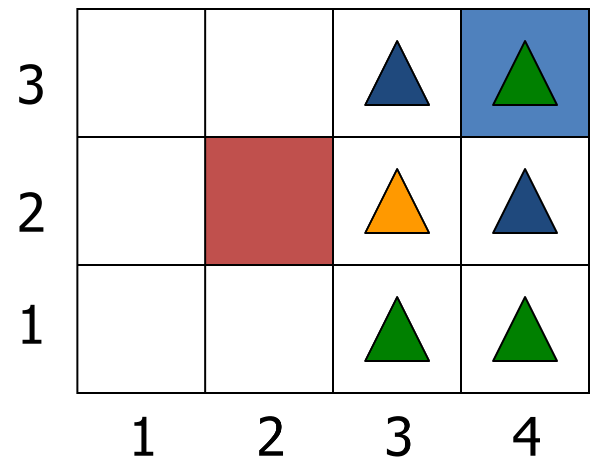

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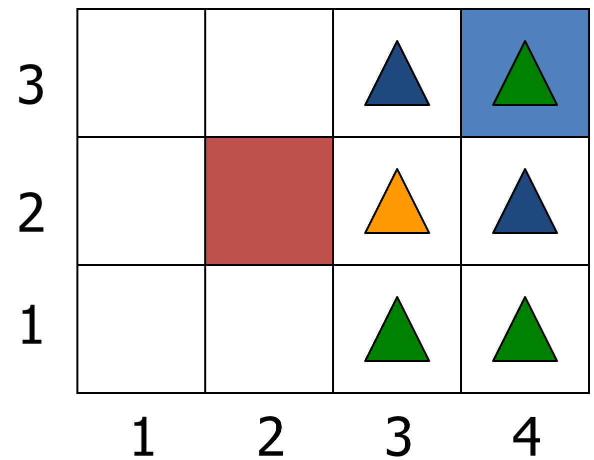

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- $\mathbf{P}([4,3] \mid (U,R).[3,2]) = 0.65$

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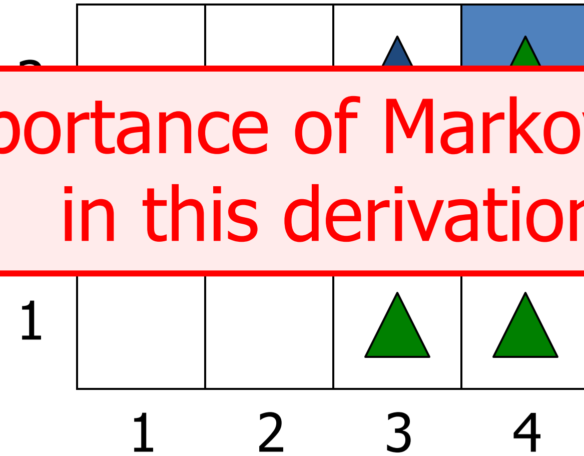

Note importance of Markov property in this derivation

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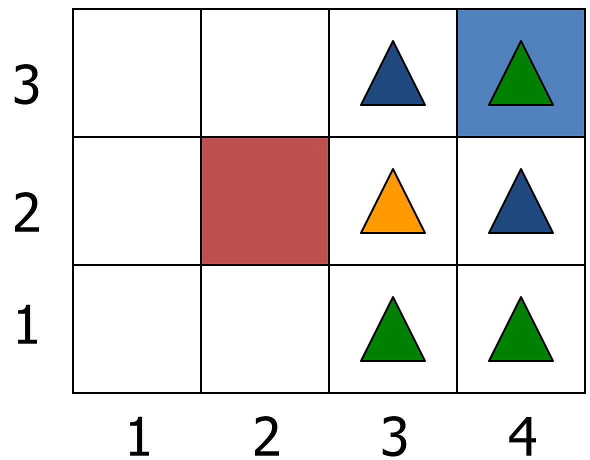
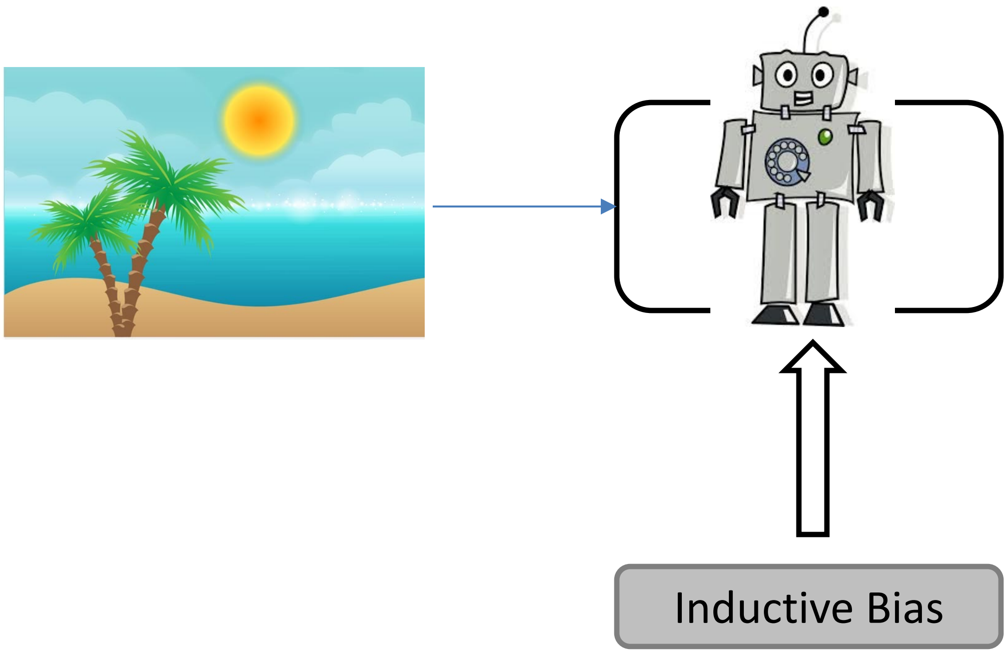
Probability of Reaching the Goal



- Main idea: **multiply backward** probabilities of each step taken from end state reached (*because* of our Markov/independence assumptions)
- But we still need to consider different ways of reaching a state
  - Going all the way around the obstacle would be "worse"

But what about the **learning** part of reinforcement learning?

RL, in our ML framework



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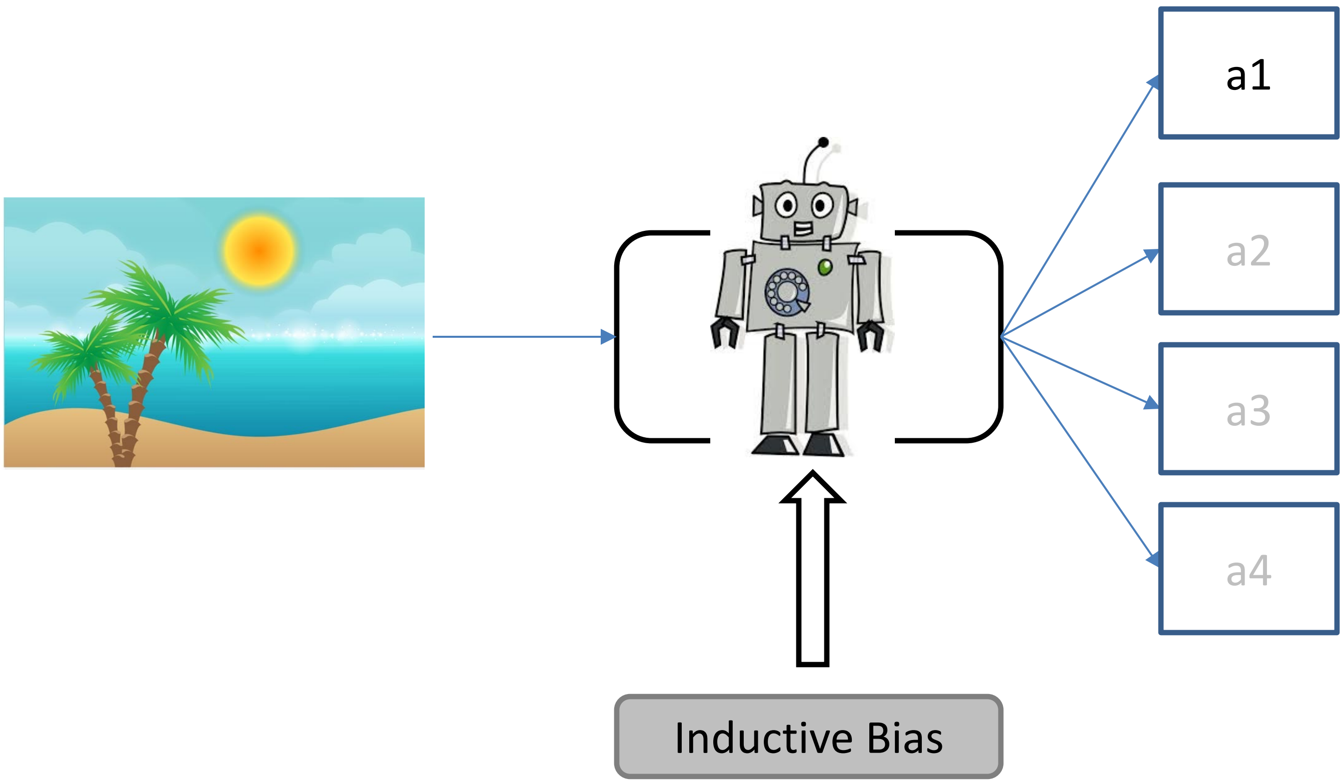

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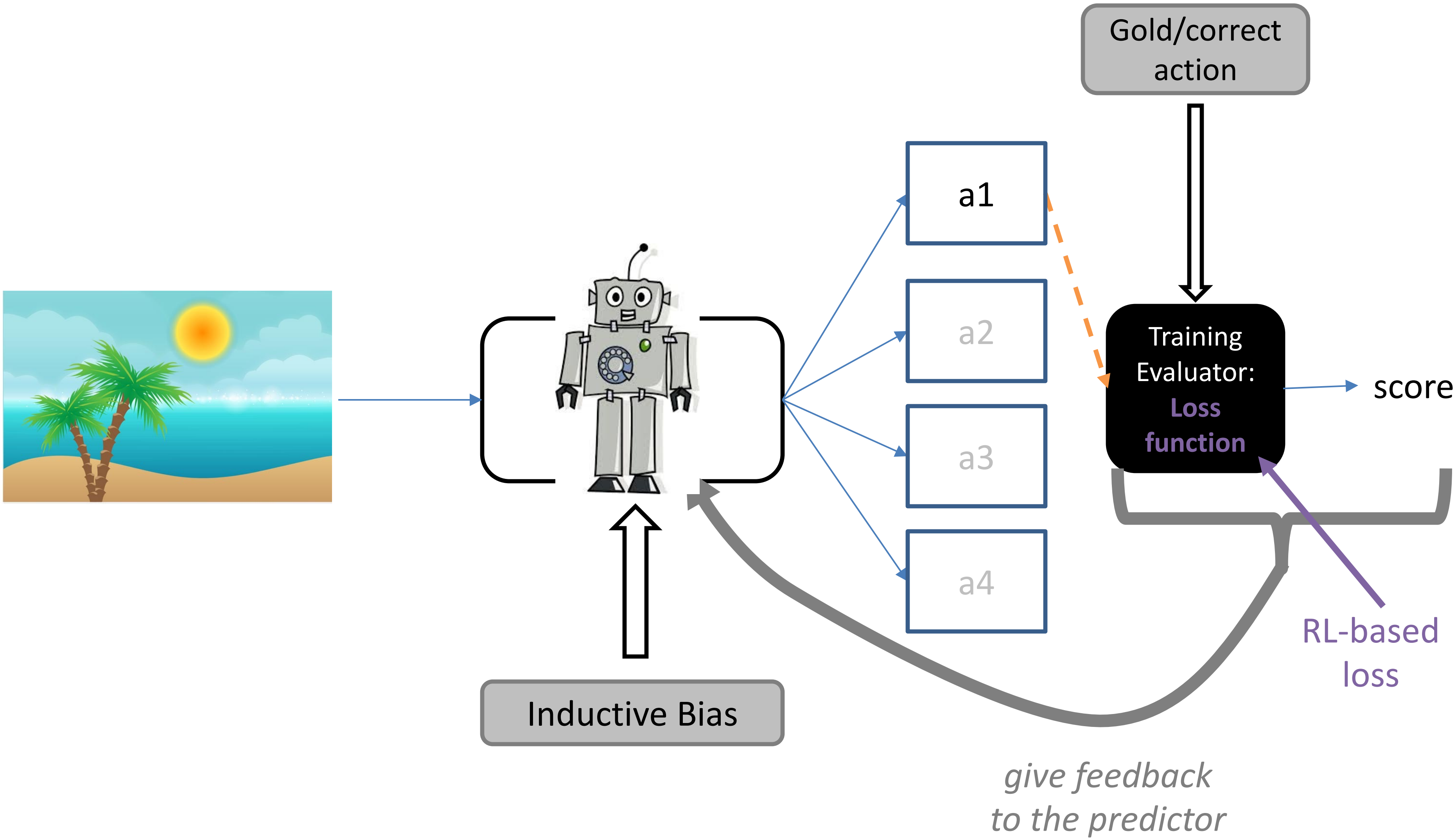
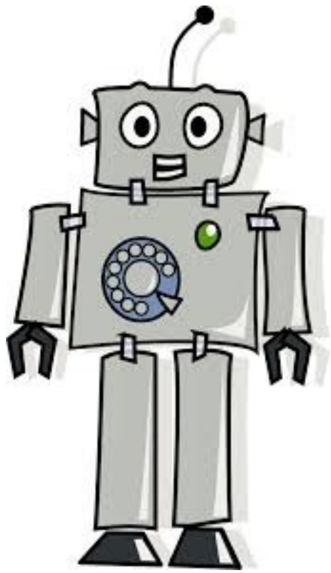
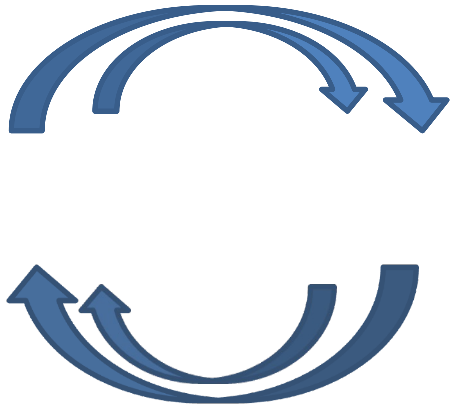
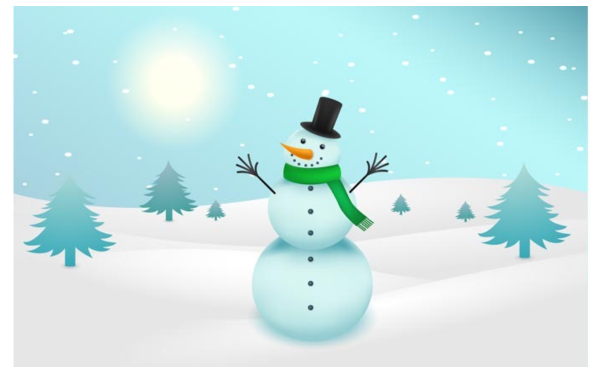


Markov Decision Process: Formalizing Reinforcement Learning

take action







get new state and/or reward

environment

agent

Markov Decision Process: $(\mathcal{S}, \mathcal{A}, \mathcal{R}, P, \gamma)$

Robot image: openclipart.org

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Markov Decision Process: Formalizing Reinforcement Learning

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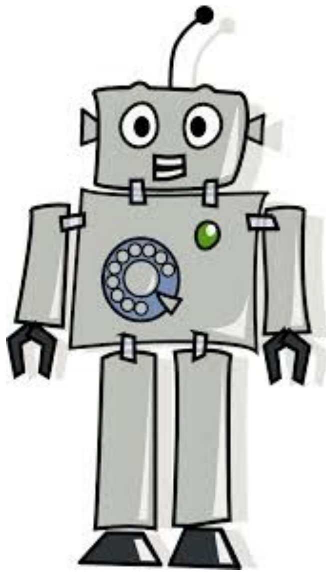

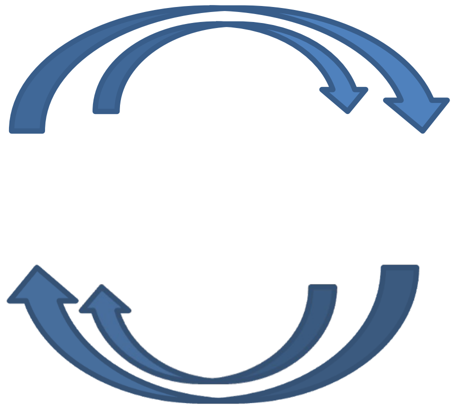

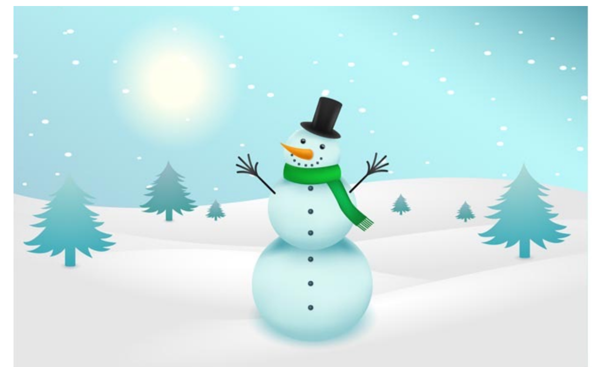

get new state and/or reward

environment

agent

set of possible actions

Markov Decision Process:

$$(\mathcal{S}, \mathcal{A}, \mathcal{R}, P, \gamma)$$

set of possible states

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Markov Decision Process: Formalizing Reinforcement Learning

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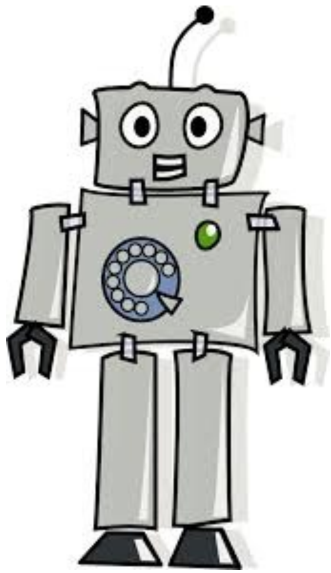

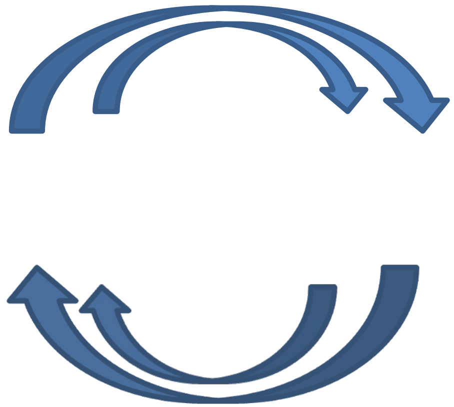

get new state
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Markov Decision Process:

set of possible actions

$$(\mathcal{S}, \mathcal{A}, \mathcal{R}, P, \gamma)$$

set of possible states

reward of (state, action) pairs

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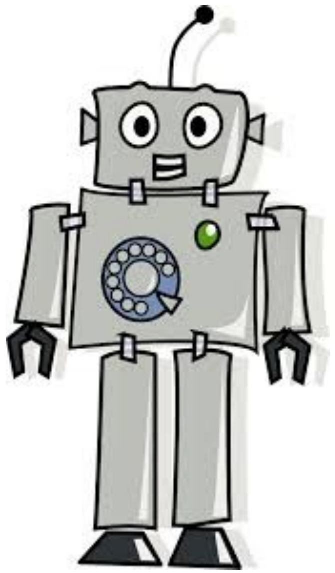
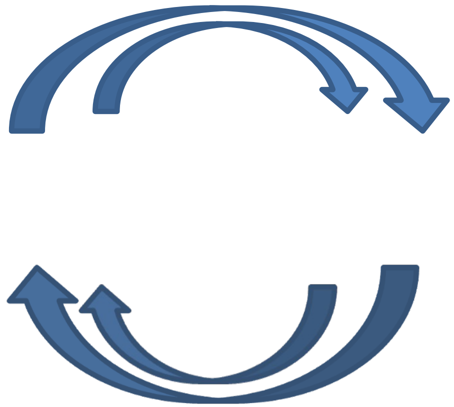
Markov Decision Process: Formalizing Reinforcement Learning

take action





get new state
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Markov Decision Process:

$$(\mathcal{S}, \mathcal{A}, \mathcal{R}, P, \gamma)$$

- $\mathcal{S}$: set of possible states
- $\mathcal{A}$: set of possible actions
- $\mathcal{R}$: reward of (state, action) pairs
- P: state-action transition distribution

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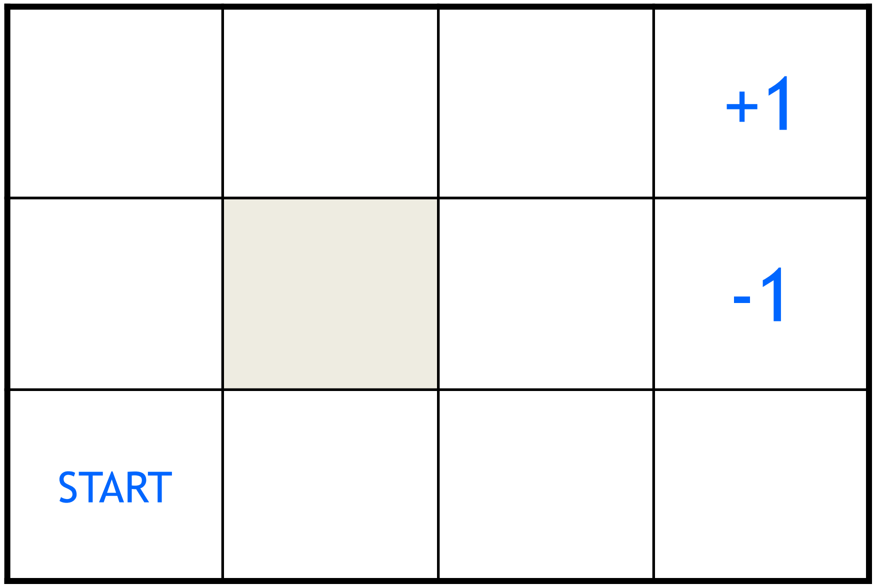
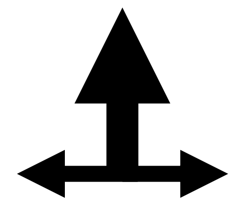
Robot in a room



actions: UP, DOWN, LEFT, RIGHT

UP

80%	move UP
10%	move LEFT
10%	move RIGHT

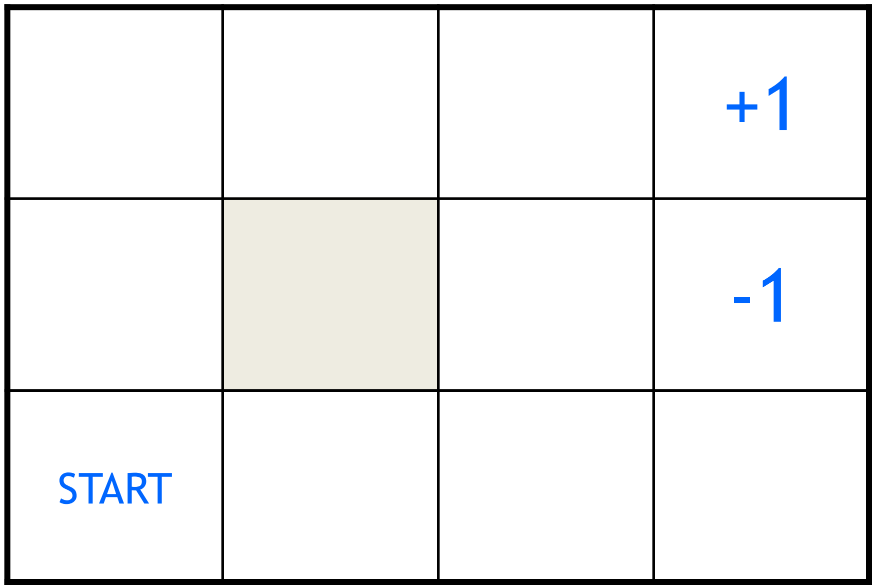


reward +1 at [4,3], -1 at [4,2]
reward -0.04 for each step

Goal: what's the strategy to achieve the maximum reward?

Slide courtesy Peter Bodík

Robot in a room



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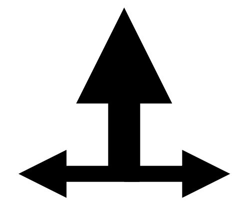

reward +1 at [4,3], -1 at [4,2]
reward -0.04 for each step

states: current location

actions: where to go next

rewards

what is the solution? Learn a mapping from (state, action) pairs to new states

Markov Decision Process: Formalizing Reinforcement Learning

Markov Decision Process:

set of possible actions

state-action transition distribution

$$(\mathcal{S}, \mathcal{A}, \mathcal{R}, P, \gamma)$$

set of possible states

reward of (state, action) pairs

discount factor

Start in initial state s_0

Markov Decision Process: Formalizing Reinforcement Learning

set of possible actions

state-action transition distribution

Markov Decision Process: $(\mathcal{S}, \mathcal{A}, \mathcal{R}, P, \gamma)$

set of possible states

reward of (state, action) pairs

discount factor

Start in initial state s_0
for t = 1 to …:
  choose action a_t

Markov Decision Process: Formalizing Reinforcement Learning

Markov Decision Process: $(\mathcal{S}, \mathcal{A}, \mathcal{R}, P, \gamma)$

- $\mathcal{S}$: set of possible states
- $\mathcal{A}$: set of possible actions
- $\mathcal{R}$: reward of (state, action) pairs
- P: state-action transition distribution
- γ: discount factor

Start in initial state s_0
for t = 1 to …:
  choose action a_t
  "move" to next state $s_t \sim \pi(\cdot | s_{t-1}, a_t)$

$Policy$
π: S → A

Markov Decision Process: Formalizing Reinforcement Learning

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- $\mathcal{R}$: reward of (state, action) pairs
- P: state-action transition distribution
- γ: discount factor

Start in initial state s_0
for t = 1 to …:
  choose action a_t
  "move" to next state $s_t \sim \pi(\cdot | s_{t-1}, a_t)$
  get reward $r_t = \mathcal{R}(s_t, a_t)$

Markov Decision Process: Formalizing Reinforcement Learning

Markov Decision Process: $(\mathcal{S}, \mathcal{A}, \mathcal{R}, P, \gamma)$

- $\mathcal{S}$: set of possible states
- $\mathcal{A}$: set of possible actions
- $\mathcal{R}$: reward of (state, action) pairs
- P: state-action transition distribution
- γ: discount factor

Start in initial state s_0
for t = 1 to …:
  choose action a_t
  "move" to next state $s_t \sim \pi(\cdot | s_{t-1}, a_t)$
  get reward $r_t = \mathcal{R}(s_t, a_t)$

objective: choose action over time to maximize time-discounted reward

Markov Decision Process: Formalizing Reinforcement Learning

set of possible actions

state-action transition distribution

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set of possible states

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Consider all possible future times t

Reward at time t

Markov Decision Process: Formalizing Reinforcement Learning

set of possible actions

state-action transition distribution

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objective: maximize discounted reward

$$\max_{\pi} \sum_{t>0} \gamma^t r_t$$

- $\sum_{t>0}$: Consider all possible future times t
- γ^t: Discount at time t
- r_t: Reward at time t

Example of Discounted Reward

objective: maximize discounted reward

$$\max_{\pi} \sum_{t>0} \gamma^t r_t$$

Consider all possible future times t

Discount at time t

Reward at time t

- If the discount factor $\gamma = 0.8$ then reward $0.8^0 r_0 + 0.8^1 r_1 + 0.8^2 r_2 + 0.8^3 r_3 + \cdots + 0.8^n r_n + \ \ldots$
- Allows you to consider all possible rewards in the future but preferring current vs. future self

Markov Decision Process: Formalizing Reinforcement Learning

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"solution": the policy π^* that maximizes the expected (average) time-discounted reward

Markov Decision Process: Formalizing Reinforcement Learning

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"solution" $$\pi^* = \operatorname*{argmax}_{\pi} \mathbb{E}\left[\sum_{t>0} \gamma^t r_t ; \pi\right]$$

Expected Value of a Random Variable

random variable

$$X \sim p(\cdot)$$

Expected Value of a Random Variable

random variable

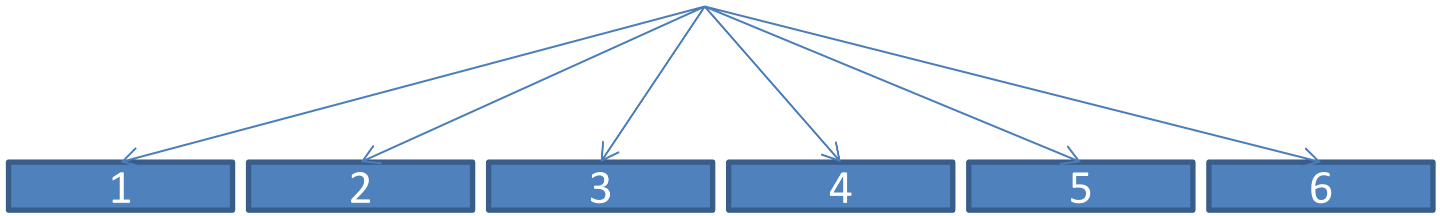
$$X \sim p(\cdot)$$

$$\mathbb{E}[X] = \sum_{x} x\, p(x)$$

expected value (distribution p is implicit)

Expected Value: Example

uniform distribution of number of cats I have



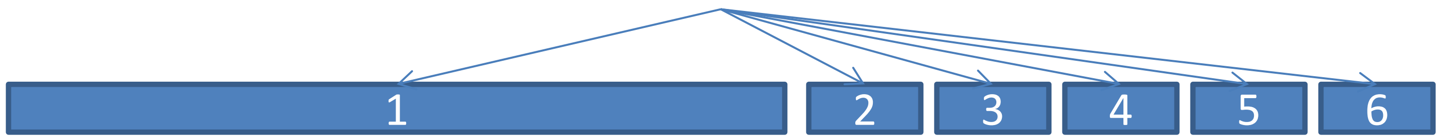
$$\mathbb{E}[X] = \sum_x x \, p(x)$$

1/6 * 1 +
1/6 * 2 +
1/6 * 3 +
1/6 * 4 +
1/6 * 5 +
1/6 * 6

= 3.5

Expected Value: Example 2

non-uniform distribution of number of cats a normal cat person has



$$\mathbb{E}[X] = \sum_{x} x \, p(x)$$

1/2 * 1 +
1/10 * 2 +
1/10 * 3 +
1/10 * 4 +
1/10 * 5 +
1/10 * 6

= 2.5

Expected Value of a Function of a Random Variable

$$X \sim p(\cdot)$$

$$\mathbb{E}[X] = \sum_{x} x\, p(x)$$

$$\mathbb{E}[f(X)] = ???$$

Expected Value of a Function of a Random Variable

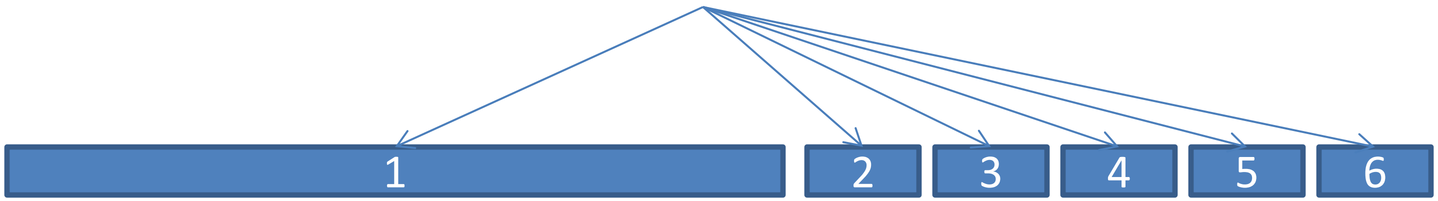
$$X \sim p(\cdot)$$

$$\mathbb{E}[X] = \sum_x x\, p(x)$$

$$\mathbb{E}[f(X)] = \sum_x f(x)\, p(x)$$

Expected Value of Function: Example

non-uniform distribution of number of cats I start with



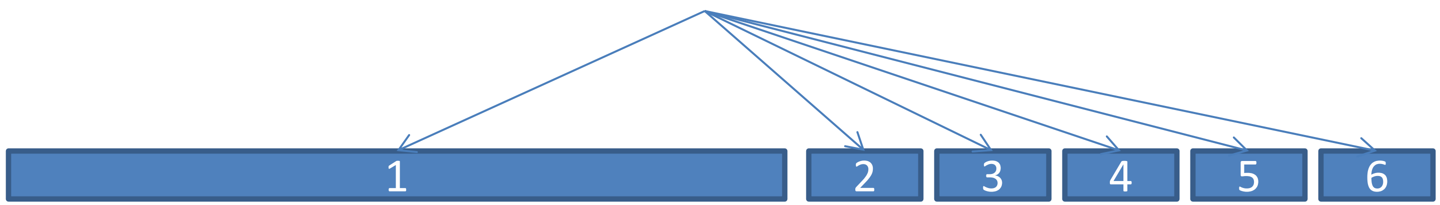
What if each cat magically becomes two?

$$f(k) = 2^k$$

$$\mathbb{E}[f(X)] = \sum_x f(x)\, p(x)$$

Expected Value of Function: Example

non-uniform distribution of number of cats I start with



What if each cat magically becomes two?

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$$\begin{aligned} &1/2 * 2^1 + \\ &1/10 * 2^2 + \\ &1/10 * 2^3 + \\ &1/10 * 2^4 + \\ &1/10 * 2^5 + \\ &1/10 * 2^6 \end{aligned} \quad = 13.4$$

Markov Decision Process: Formalizing Reinforcement Learning

Mar

Here, r_t is a function of random variable s_t.

Start in initia

for t = 1 to ..

choose act

"move" to next state $s_t \sim \pi(\cdot | s_{t-1}, a_t)$

get reward $r_t = \mathcal{R}(s_t, a_t)$

$$\max_{\pi} \sum_{t>0} \gamma^t r_t$$

"solution" $\pi^* = \underset{\pi}{\operatorname{argmax}}\, \mathbb{E}\left[\sum_{t>0} \gamma^t r_t \,; \pi\right]$

Markov Decision Process: Formalizing Reinforcement Learning

Mar

Here, r_t is a function of random variable s_t. ➔
The expectation is over the different states s_t the agent could be in at time *t* (equiv. actions the agent could take).

Start in initia
for t = 1 to ..
choose ac
"move" to next state $s_t \sim \pi(\cdot|s_{t-1}, a_t)$
get reward $r_t = \mathcal{R}(s_t, a_t)$

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Some Challenges

1. Representing states (and actions)

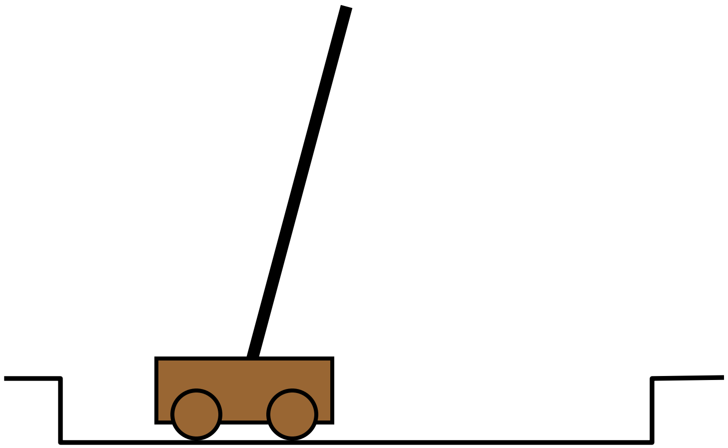
2. Defining our reward

3. Learning our policy

State Representation

Task: pole-balancing

state representation?



move car left/right to
keep the pole balanced

State Representation

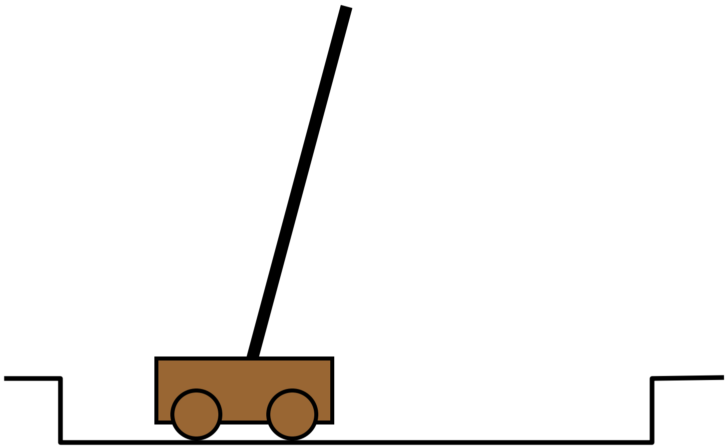
Task: pole-balancing

state representation

position and velocity of car

angle and angular velocity of pole

what about *Markov property*?



move car left/right to
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State Representation

Task: pole-balancing

state representation

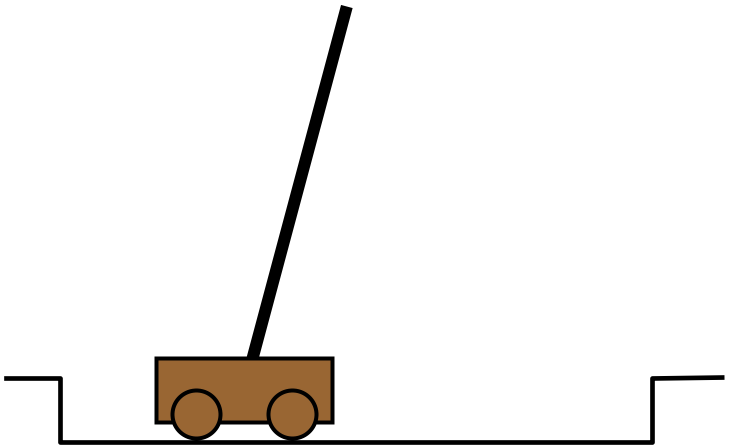
position and velocity of car

angle and angular velocity of pole

what about *Markov property*?

would need more info

noise in sensors, temperature, bending of pole



move car left/right to keep the pole balanced

Some Challenges

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Designing Rewards

robot in a maze

- episodic task, not discounted, +1 when out, 0 for each step

chess

- GOOD: +1 for winning, -1 losing
- BAD: +0.25 for taking opponent's pieces
  - high reward even when lose

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- rewards indicate what we want to accomplish
- NOT how we want to accomplish it

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rewards

- rewards indicate what we want to accomplish
- NOT how we want to accomplish it

shaping

- positive reward often very "far away"
- rewards for achieving subgoals (domain knowledge)
- also: adjust initial policy or initial value function

Slide courtesy/adapted: Peter Bodík

Simple Reinforcement Learning

- Feedback is at the end, on a **series** of actions.
- Very early concept in Artificial Intelligence!
- Arthur Samuels' checker program was a simple reinforcement based learner, initially developed in 1956.
- In 1962 it beat a human checkers master.

www-03.ibm.com/ibm/history/ibm100/us/en/icons/ibm700series/impacts/

Example: Animals Game

– Human: I’m thinking of an animal.

Example: Animals Game

- Human: I’m thinking of an animal.
- Computer: Is it a bird?

Example: Animals Game

– Human: I'm thinking of an animal.
– Computer: Is it a bird?
– Human: No.

Example: Animals Game

– Human: I’m thinking of an animal.
– Computer: Is it a bird?
– Human: No.
– Computer: Is it a frog?

Example: Animals Game

- Human: I’m thinking of an animal.
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- Computer: Is it a frog?
- Human: No.
- Computer: What is it?

Example: Animals Game

- Human: I’m thinking of an animal.
- Computer: Is it a bird?
- Human: No.
- Computer: Is it a frog?
- Human: No.
- Computer: What is it?
- Human: A mouse.

Example: Animals Guessing Game Architecture

- All of the parts of ML Architecture:
  - The Representation is a sequence of questions and pairs of yes/no answers (decision tree).
  - The Actor “walks” the tree, interacting with a human; at each question it chooses whether to follow the “yes” branch or the “no” branch.
  - The Evaluator/loss function is the human player telling the game whether it has guessed correctly.
  - The Learner elicits new questions and adds questions, guesses and branches to the tree.

Reinforcement Learning (cont.)

- Goal: agent acts in the world to maximize its rewards
- Agent has to figure out what it did that made it get that reward/punishment
  - This is known as the credit assignment problem

Example: Animals Game

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Example: Animals Game

- Human: I’m thinking of an animal.
- Computer: Is it a bird?
- Human: No.
- Computer: Is it a frog?
- Human: No.
- Computer: What is it?
- Human: A mouse.
- Computer: **Tell me a question that distinguishes a frog from a mouse.**

Example: Animals Game

- Human: I’m thinking of an animal.
- Computer: Is it a bird?
- Human: No.
- Computer: Is it a frog?
- Human: No.
- Computer: What is it?
- Human: A mouse.
- Computer: **Tell me a question that distinguishes a frog from a mouse.**
- Human: Does it have fur?

Example: Animals Game

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- Human: Does it have fur?
- Computer: What is the answer for a frog?

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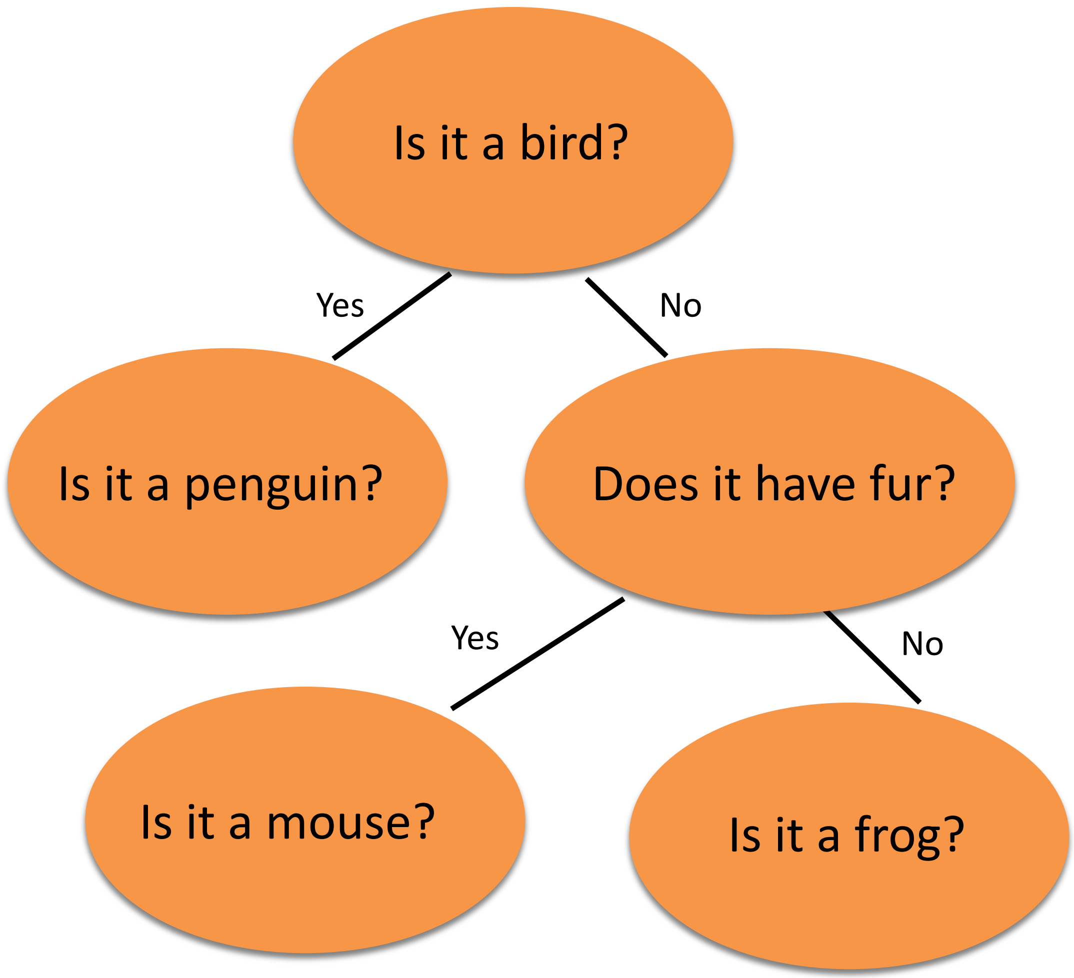
- Computer: What is it?
- Human: A mouse.
- Computer: **Tell me a question that distinguishes a frog from a mouse.**
- Human: Does it have fur?
- Computer: What is the answer for a frog?
- Human: No.

Example: Animals Behind the Scene

Computer: Is it a bird?
Human: No
Computer: Is it a frog?
Human: No
Computer: What is it?
Human: A mouse
Computer: Tell me a question that distinguishes a frog from a mouse.
Human: Does it have fur?
Computer: What is the answer for a frog?
Human: no



After several rounds...

Reinforcement Learning (cont.)

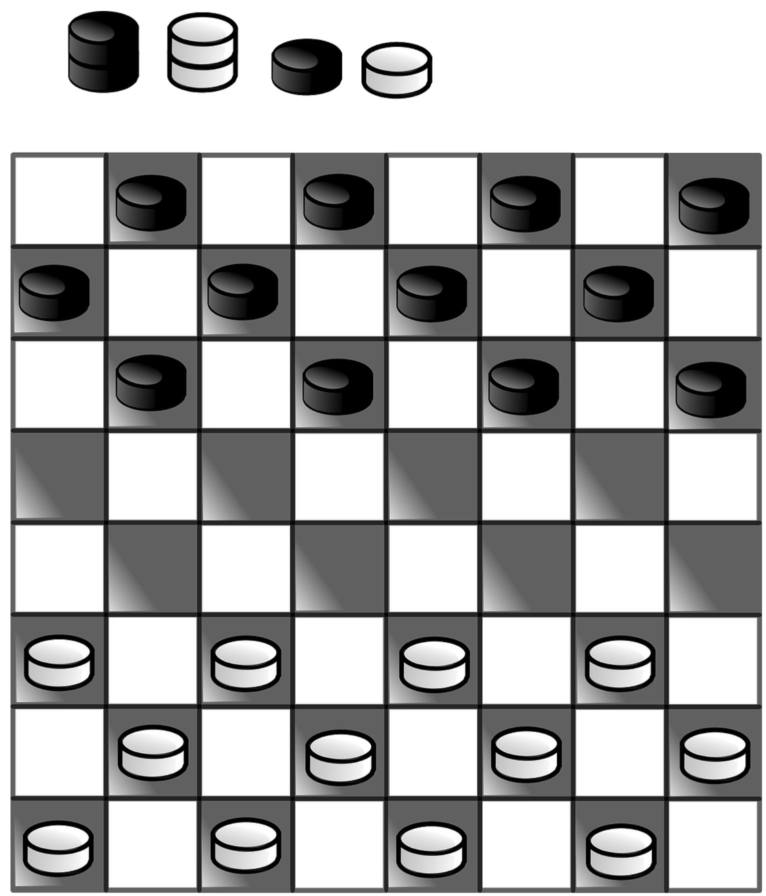
- Goal: agent acts in the world to maximize its rewards
- Agent has to figure out what it did that made it get that reward/punishment
  - This is known as the credit assignment problem
- RL can be used to train computers to do many tasks
  - Backgammon and chess playing
  - Job shop scheduling
  - Controlling robot limbs

Reactive Agent

- This kind of agent is a reactive agent
- The general algorithm for a reactive agent is:
  - Observe some state
  - If it is a terminal state, stop
  - Otherwise choose an action from the actions possible in that state
  - Perform the action
  - Recur.

Simple Example

- Learn to play checkers
  - Two-person game
  - 8x8 boards, 12 checkers/side
  - relatively simple set of rules: http://www.darkfish.com/checkers/rules.html
  - Goal is to eliminate all your opponent's pieces



https://pixabay.com/en/checker-board-black-game-pattern-29911

Representing Checkers

- First we need to represent the game
- To completely describe one step in the game you need
  – A representation of the game board.
  – A representation of the current pieces
  – A variable which indicates whose turn it is
  – A variable which tells you which side is "black"
- There is no history needed
- A look at the current board setup gives you a complete picture of the state of the game

Representing Checkers

- Second, we need to represent the rules
- Represented as a **set of allowable moves** given board state
  - If a checker is at row x, column y, and row x+1 column y±1 is empty, it can move there.
  - If a checker is at (x,y), a checker of the opposite color is at (x+1, y+1), and (x+2,y+2) is empty, the checker must move there, and remove the "jumped" checker from play.
- There are additional rules, but all can be expressed in terms of the state of the board and the checkers.
- Each rule includes the outcome of the relevant action in terms of the state.
- What's a good reward?

A More Complex Example

- Consider an agent which must learn to drive a car
  - State?
  - Possible actions?
  - Rewards?

Some Challenges

1. Representing states (and actions)

2. Defining our reward

3. Learning our policy

Overview: Learning Strategies

Dynamic Programming

Q-learning

Monte Carlo approaches

Dynamic programming

use value functions to structure the search for good policies

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policy evaluation: compute V^{π} from π

policy improvement: improve π based on V^{π}

Dynamic programming

use value functions to structure the search for good policies

policy evaluation: compute V^{π} from π

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start with an arbitrary policy

repeat evaluation/improvement until convergence

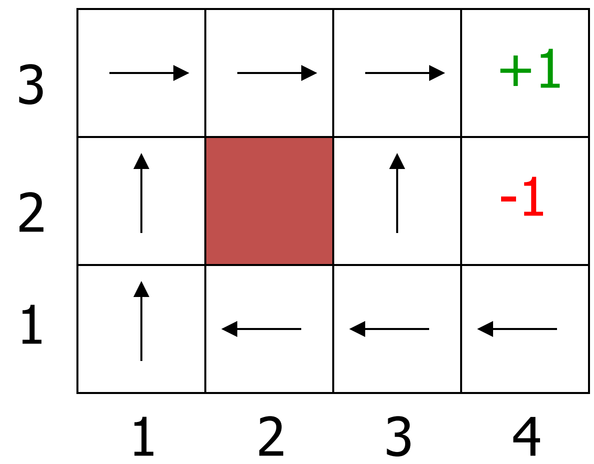
Reactive Agent Algorithm

Repeat:

Accessible or observable state

- s ← sensed state
- If s is a terminal state then exit
- a ← choose action (given s)
- Perform a

Policy (Reactive/Closed-Loop Strategy)



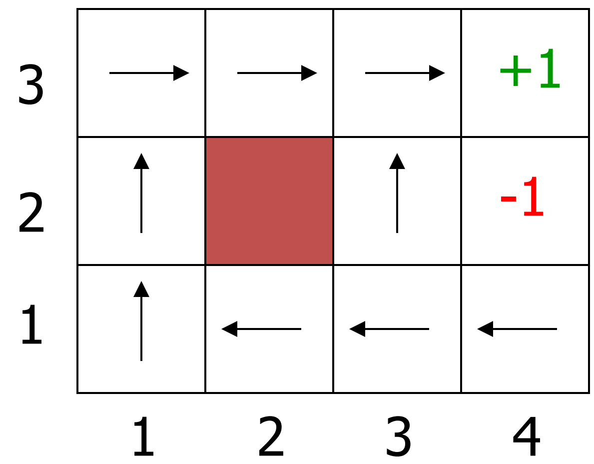
- In every state, we need to know what to do
- The **goal** doesn't change
- A policy (Π) is a complete mapping from *states* to *actions*
  - "If in [3,2], go up; if in [3,1], go left; if in…"

Reactive Agent Algorithm

Repeat:

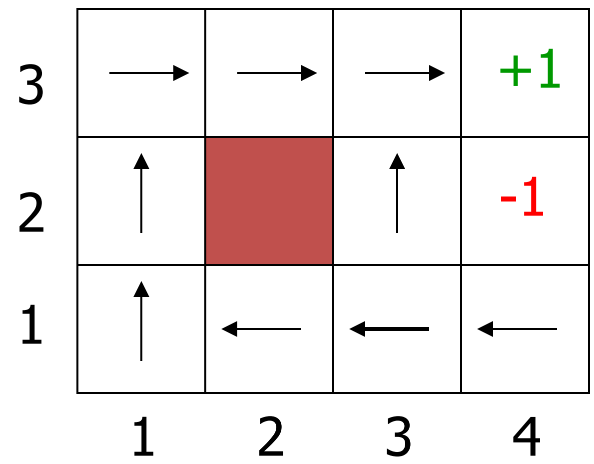
- s ← sensed state
- If s is terminal then exit
- a ← Π(s)
- Perform a

Optimal Policy



- A policy Π is a complete mapping from states to actions
- The optimal policy Π^* is the one that always yields a history (sequence of steps ending at a terminal state) with maximal ***expected*** utility

Optimal Policy



- A policy Π is a comp ... ns
- The optimal policy Π ... history with maximal expected utility

This problem is called a Markov Decision Problem (MDP)

How to compute Π*?

Defining Value Function

- Problem:
  - When making a decision, we only know the reward so far, and the possible actions

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  - What is the value function of a particular ***state*** in the middle of decision making?
  - Need to compute ***expected value function*** of possible future histories/states

Defining Value Function

$$V^{\pi}(s) = \mathrm{E}\left[R(s_0) + \gamma R(s_1) + \gamma^2 R(s_2) + \cdots \,\middle|\, s_0 = s, \pi\right].$$

$V^{\pi}(s)$ is simply the expected sum of discounted rewards upon starting in state s, and taking actions according to π.[1]

Given a fixed policy π, its value function V^{π} satisfies the **Bellman equations**:

$$V^{\pi}(s) = R(s) + \gamma \sum_{s' \in S} P_{s\pi(s)}(s') V^{\pi}(s').$$

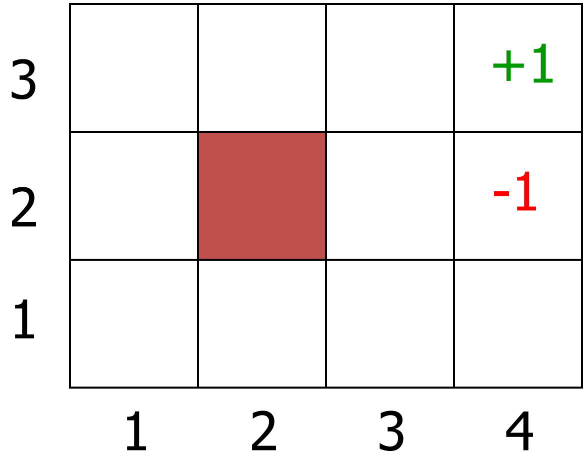
- What is the value function of a particular ***state*** in the middle of decision making?
- Need to compute ***expected value function*** of possible future histories/states

Value Iteration

Algorithm 4 Value Iteration

1: For each state s, initialize $V(s) := 0$.
2: **for** until convergence **do**
3: For every state, update

$$V^*(s) = R(s) + \max_{a \in A} \gamma \sum_{s' \in S} P_{sa}(s') V^*(s'). \tag{15.4}$$



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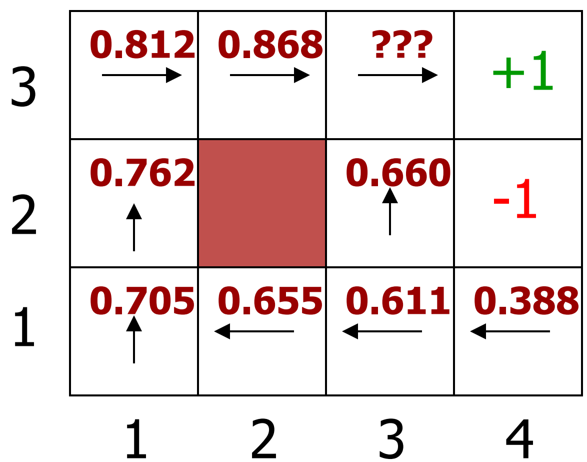

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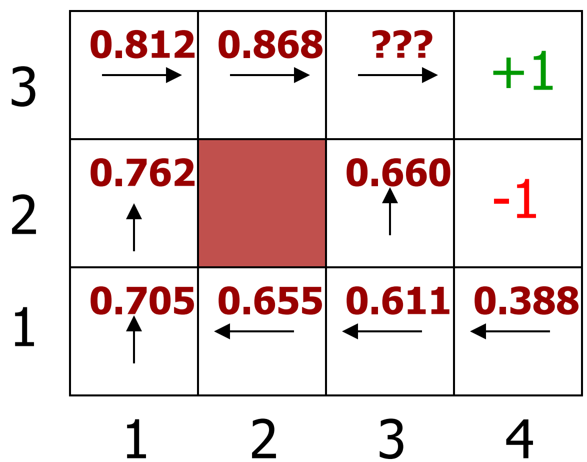

EXERCISE: What is V*([3,3]) (assuming that the other V* are as shown)?

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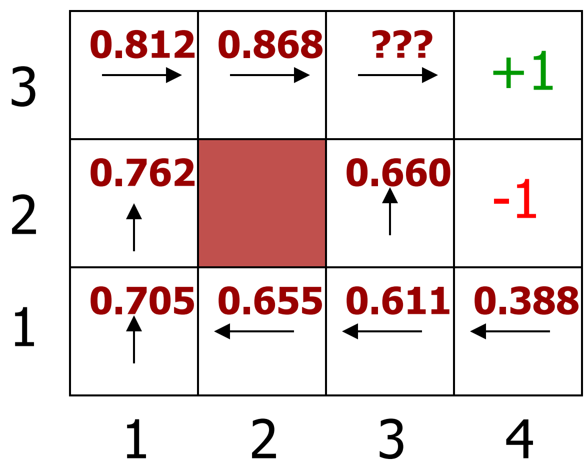

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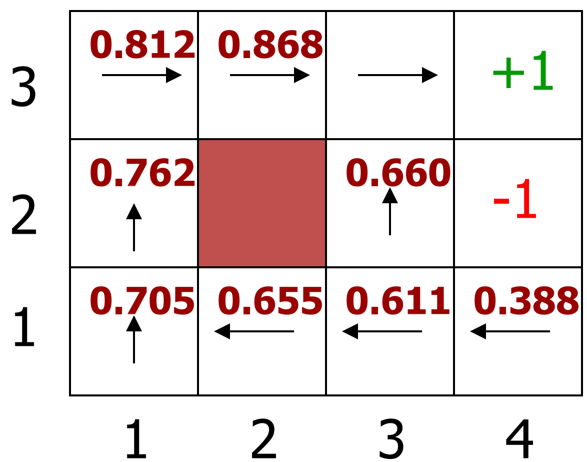

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Value Iteration

Algorithm 4 Value Iteration

For each state s, initialize $V(s) := 0$.

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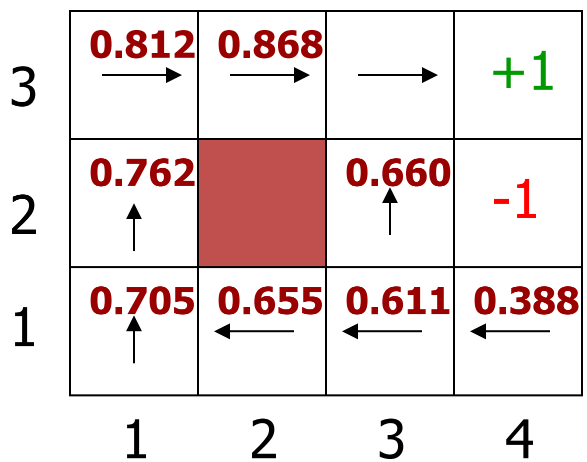

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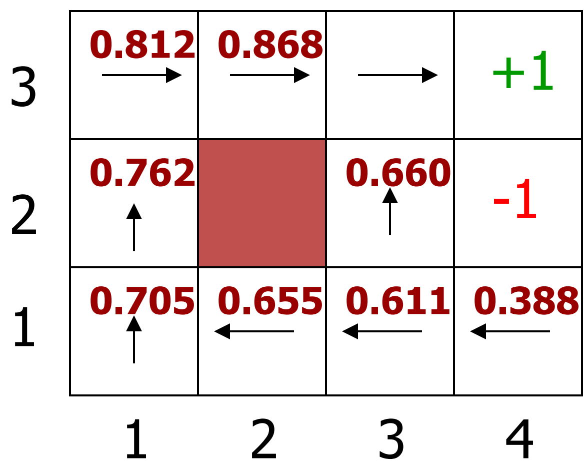

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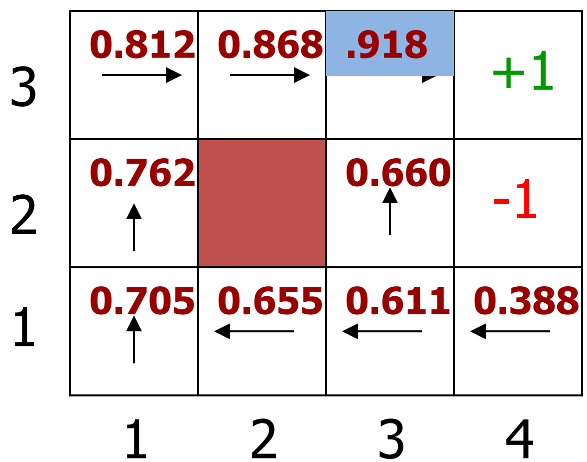

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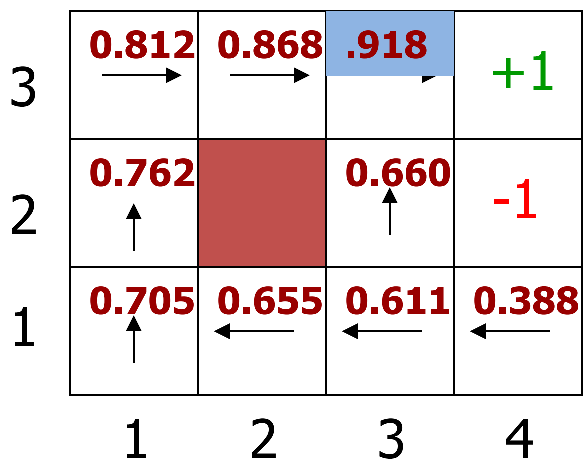

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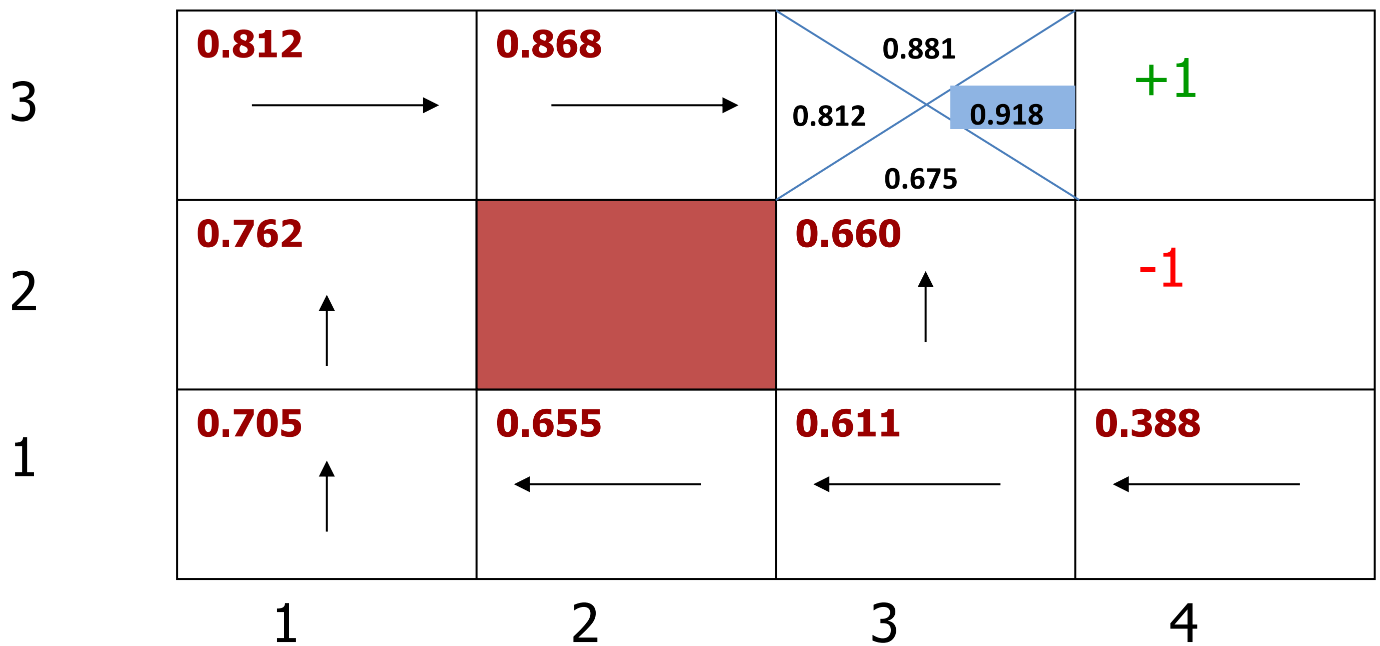
More Breakdown

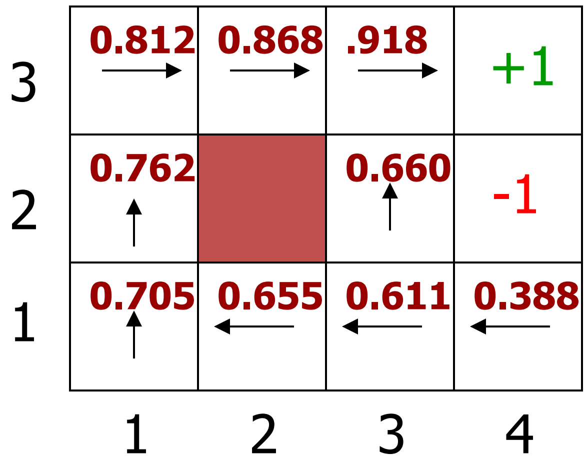
Value Iteration

In (3, 3), since → action gave us the **maximum expected future reward,** we choose to keep → in our policy. Same thing was done for all states.



Optimal Policy

$$\pi^*(s) = \arg\max_{a \in A} \sum_{s' \in S} P_{sa}(s') V^*(s').$$



Whichever is higher becomes next action for (3, 1)

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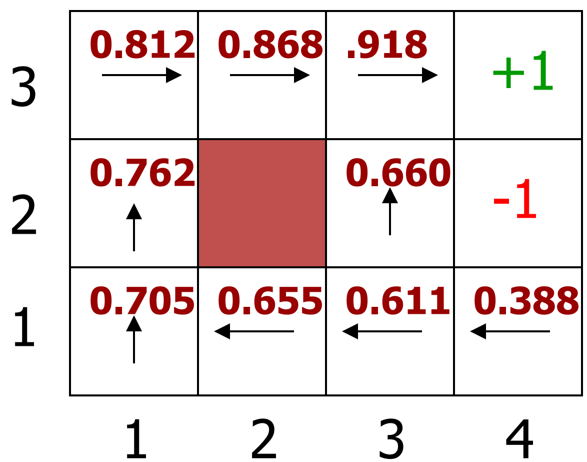
$\pi^*_{3,1}$ being (←) =
$P_{up}\, V^*_{2,1} + P_{left}\, V^*_{3,1}$ (Bounced off) $+ P_{right}\, V^*_{3,2}$
= 0.8 * 0.655 + 0.1 * 0.611 + 0.1 * 0.66



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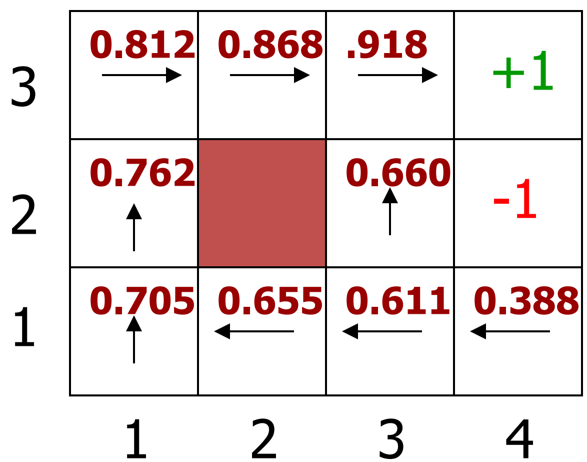

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Whichever is higher becomes next action for (3, 1)

Policy Iteration

- Pick a policy Π at random
- Repeat:
  - Compute Value function of each state for Π

$$V(s) := V^{\pi}(s) = R(s) + \gamma \sum_{s' \in S} P_{s\pi(s)}(s') V^{\pi}(s').$$

  - Compute the policy Π' given these value functions

$$\pi'(s) := \arg\max_{a \in A} \sum_{s'} P_{sa}(s') V(s').$$

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Or solve the set of linear equations:
(often a sparse system)

$$\pi'(s) := \arg\max_{a \in A} \sum_{s'} P_{sa}(s') V(s').$$

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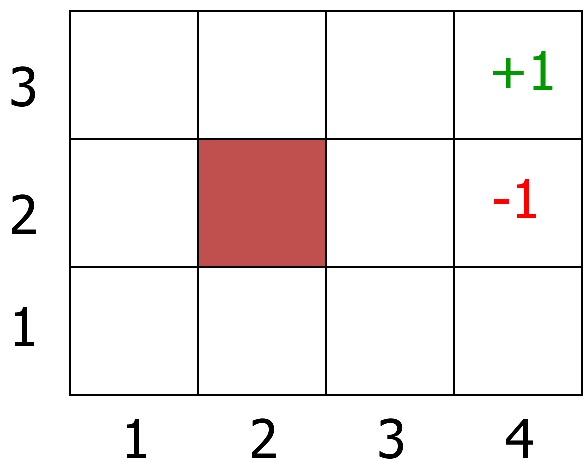
Advanced topic

Infinite Horizon

In many problems, e.g., the robot navigation example, histories are potentially unbounded and the same state can be reached many times



Advanced topic

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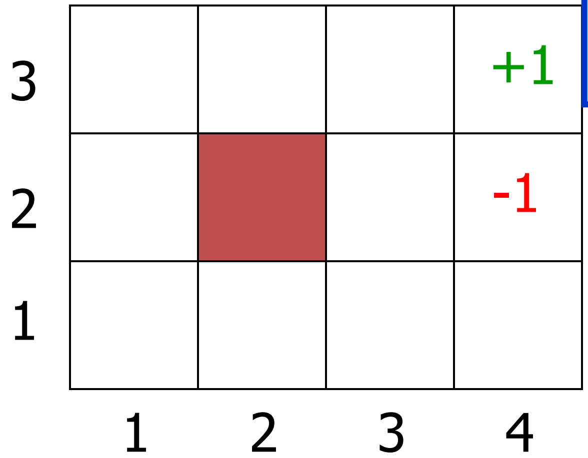

What if the robot lives forever?

Advanced topic

Infinite Horizon

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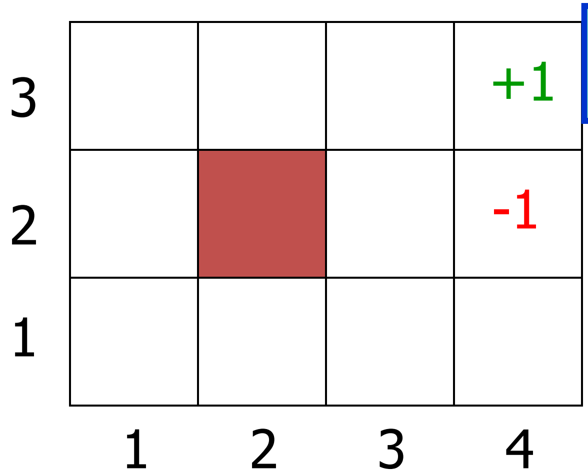

What if the robot lives forever?

One trick:
Use discounting to make an infinite horizon problem mathematically tractable

Value Iteration: Summary

- Initialize state values (expected utilities) randomly
- Repeatedly update state values using best action, according to current approximation of state values
- Terminate when state values stabilize
- Resulting policy will be the best policy because it’s based on accurate state value estimation

Policy Iteration: Summary

– Initialize policy randomly
– Repeatedly update state values using best action, according to current approximation of state values
– Then update policy based on new state values
– Terminate when policy stabilizes
– Resulting policy is the best policy, but state values may not be accurate (may not have converged yet)
– Policy iteration is often faster (because we don't have to get the state values right)

- **Both methods have a major weakness: They require us to know the transition function exactly in advance!**

Exploration vs. Exploitation

- Problem with naïve reinforcement learning:
  - What action to take?
  - **Best apparent action, based on learning to date** } Exploitation
    - Greedy strategy
    - Often prematurely converges to a suboptimal policy!
  - **Random (or unknown) action** } Exploration
    - Will cover entire state space
    - Very expensive and slow to learn!
    - When to stop being random?
  - Balance exploration (try random actions) with exploitation (use best action so far)

More on Exploration

- Agent may sometimes choose to explore suboptimal moves in hopes of finding better outcomes
  - Only by visiting all states frequently enough can we guarantee learning the true values of all the states
- When the agent is **learning**, ideal would be to get accurate values for all states
  - Even though that may mean getting a negative outcome
- When agent is **performing**, ideal would be to get optimal outcome
- A learning agent should have an **exploration policy**

Exploration Policy

- Wacky approach (exploration): act randomly in hopes of eventually exploring entire environment
  - Choose any legal checkers move
- Greedy approach (exploitation): act to maximize utility using current estimate
  - Choose moves that have in the past led to wins
- Reasonable balance: act more wacky (exploratory) when agent has little idea of environment; more greedy when the model is close to correct
  - Suppose you know no checkers strategy?
  - What’s the best way to get better?

Overview: Learning Strategies

Dynamic Programming

Q-learning

Monte Carlo approaches

Q-learning

$$Q: (s, a) \rightarrow \mathbb{R}$$

Goal: learn a function that computes a "goodness" score for taking a particular action a in state s

Q-learning

previous algorithms: on-policy algorithms

- start with a random policy, iteratively improve
- converge to optimal

Q-learning: off-policy

- use any policy to estimate Q

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha \left[r_{t+1} + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t) \right]$$

- Q directly approximates Q* (Bellman optimality equation)
- independent of the policy being followed
- only requirement: keep updating each (s,a) pair

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R(s$_t$)

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Deep/Neural Q-learning

$$Q(s, a; \theta) \approx Q^*(s, a)$$

neural network desired optimal solution

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Approach: Form (and learn) a neural network to model our optimal Q function

Deep/Neural Q-learning

Learn weights (parameters) θ of our neural network

$$Q(s, a; \theta) \approx Q^*(s, a)$$

neural network desired optimal solution

Approach: Form (and learn) a neural network to model our optimal Q function

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Monte Carlo policy evaluation

don't need full knowledge of environment (just (simulated) experience)

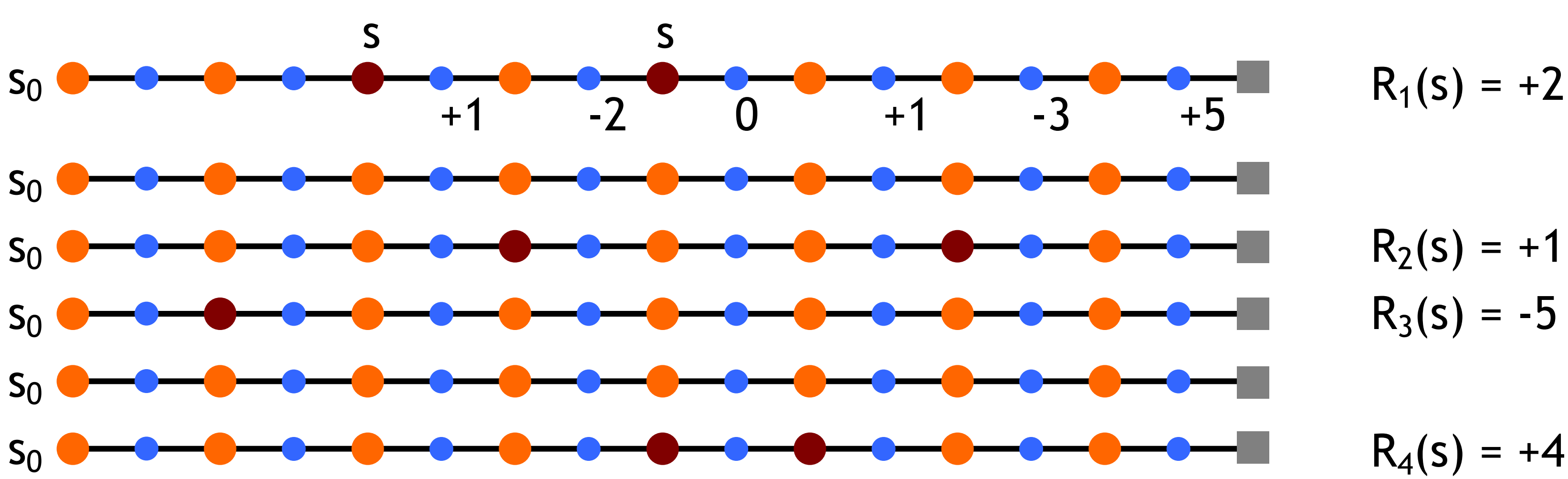
want to estimate $V^{\pi}(s)$

Monte Carlo policy evaluation

don't need full knowledge of environment (just **(simulated)** experience)

want to estimate $V^{\pi}(s)$

- expected return starting from s and following π
- estimate as average of observed returns in state s



$$V^{\pi}(s) \approx (2 + 1 - 5 + 4)/4 = 0.5$$

Maintaining exploration

key ingredient of RL

deterministic/greedy policy won't explore all actions

- don't know anything about the environment at the beginning
- need to try all actions to find the optimal one

maintain exploration

- use *soft* policies instead: $\pi(s,a)>0$ (for all s,a)

ε-greedy policy

- with probability 1-ε perform the optimal/greedy action
- with probability ε perform a random action

- will keep exploring the environment
- slowly move it towards greedy policy: ε -> 0

RL Summary 1:

- **Reinforcement learning systems**
  - Learn **series** of actions or decisions, rather than a single decision
  - Based on feedback given at the end of the series
- A reinforcement learner has
  - A goal
  - Carries out trial-and-error search
  - Finds the best paths toward that goal

RL Summary 2:

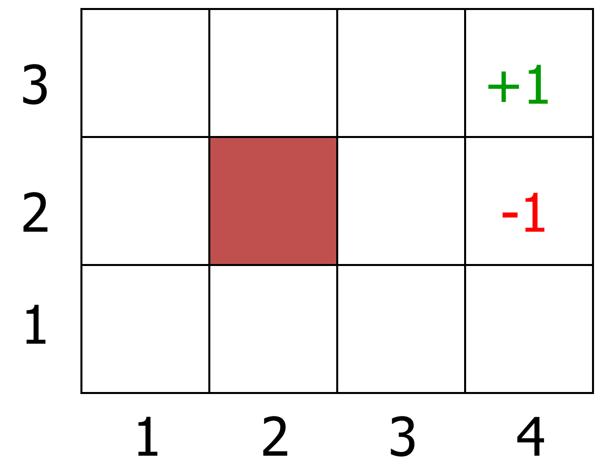
- A typical reinforcement learning system is an active agent, interacting with its environment.
- It must balance:
  - Exploration: trying different actions and sequences of actions to discover which ones work best
  - Exploitation (achievement): using sequences which have worked well so far
- Must learn **successful sequences of actions** in an uncertain environment

RL Summary 3

- Very hot area of research at the moment
- There are **many** more sophisticated RL algorithms
  - Most notably: probabilistic approaches
- Applicable to game-playing, search, finance, robot control, driving, scheduling, diagnosis, …

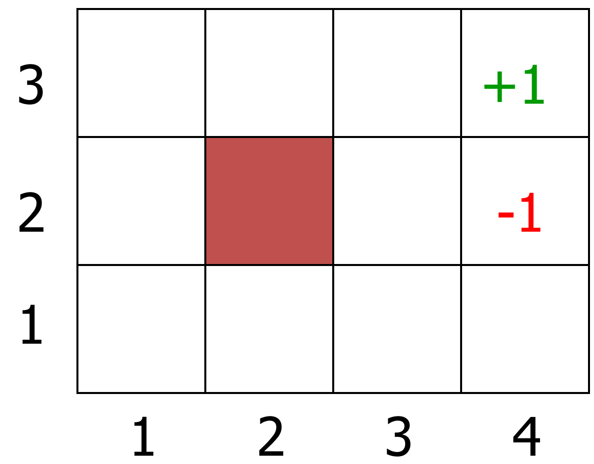
EXTRA SLIDES

Utility Function



- [4,3] provides power supply
- [4,2] is a sand area from which the robot cannot escape

Utility Function



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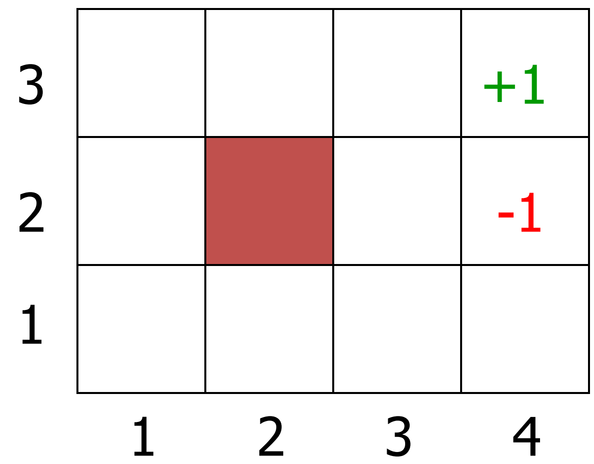

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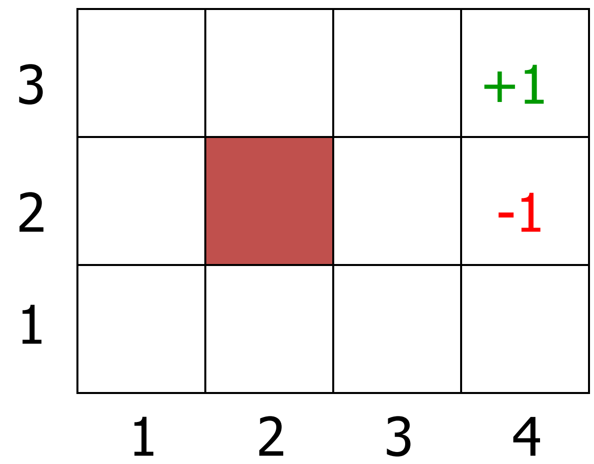

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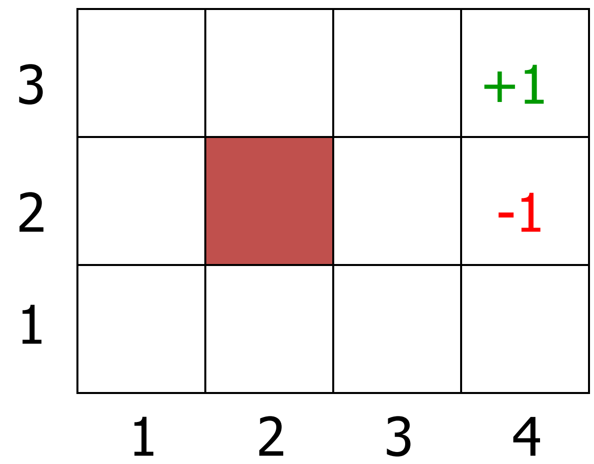
- Histories have utility!

Utility of a History



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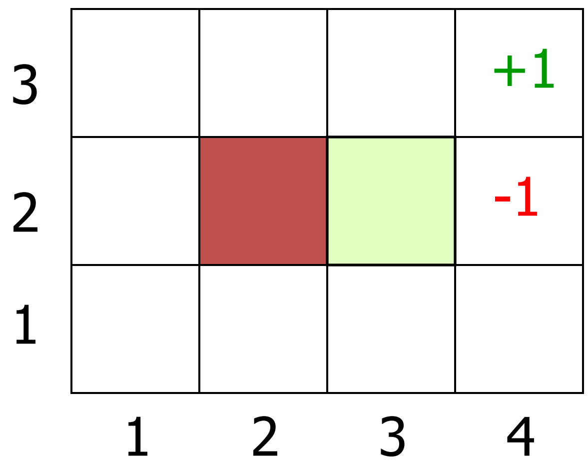
- The robot needs to recharge its batteries
- [4,3] or [4,2] are terminal states
- Histories have utility!
- The utility of a history is defined by the utility of the last state (+1 or –1) minus $n/25$, where n is the number of moves
  - Many utility functions possible, for many kinds of problems.

Utility of an Action Sequence



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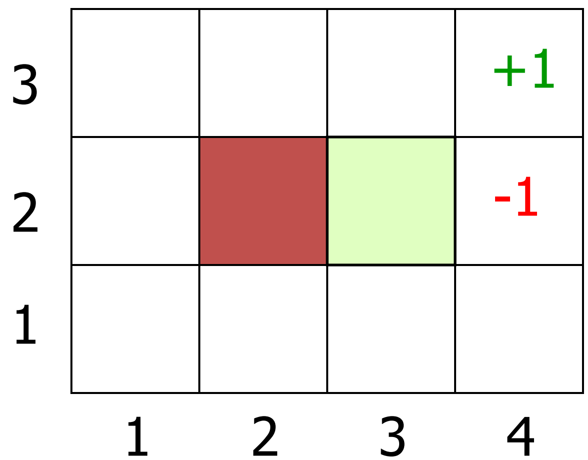
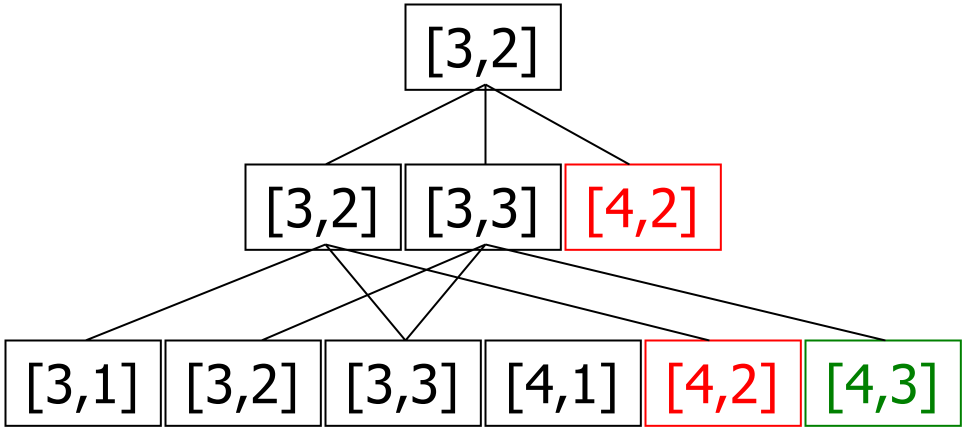
Utility of an Action Sequence





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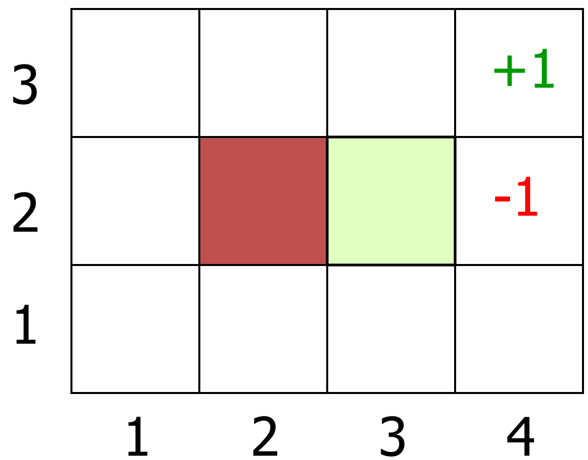
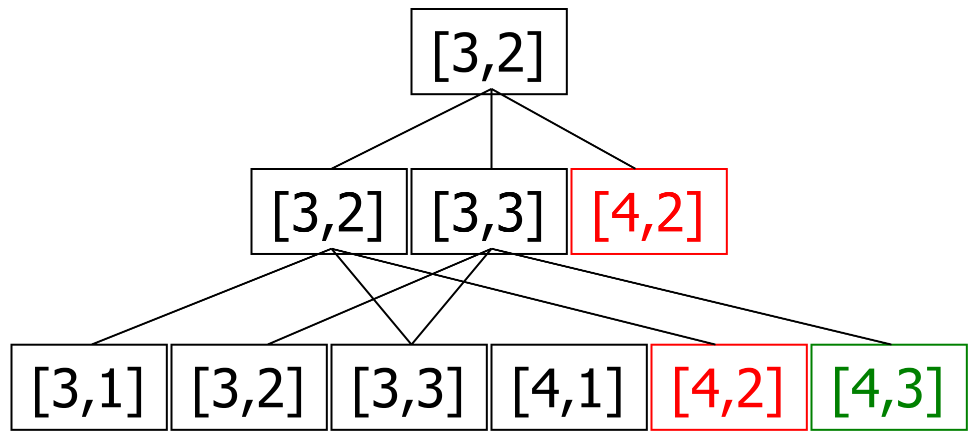
Utility of an Action Sequence





- Consider the action sequence (U,R) from [3,2]
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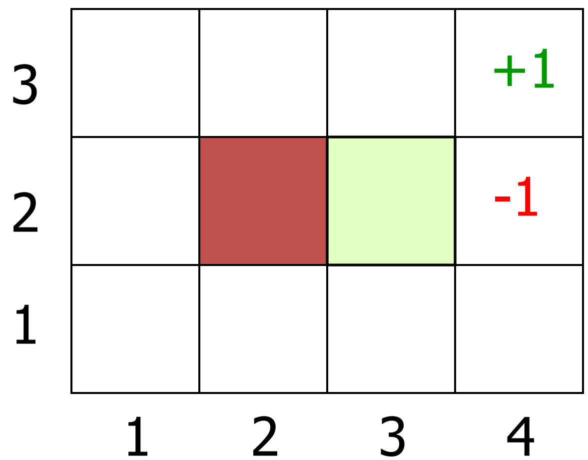
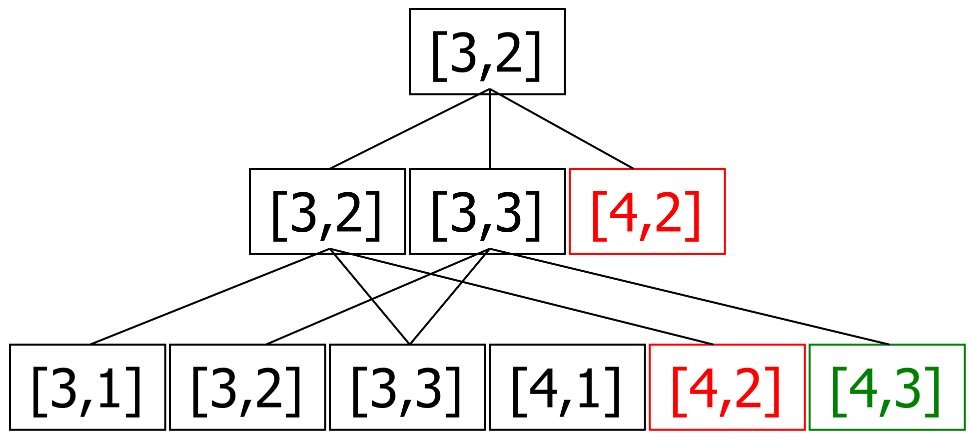
$$\mathcal{U} = \Sigma_h \mathcal{U}_h \mathbf{P}(h)$$

Optimal Action Sequence





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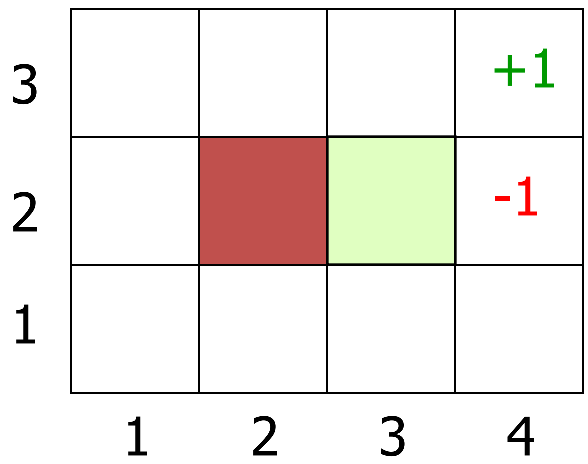

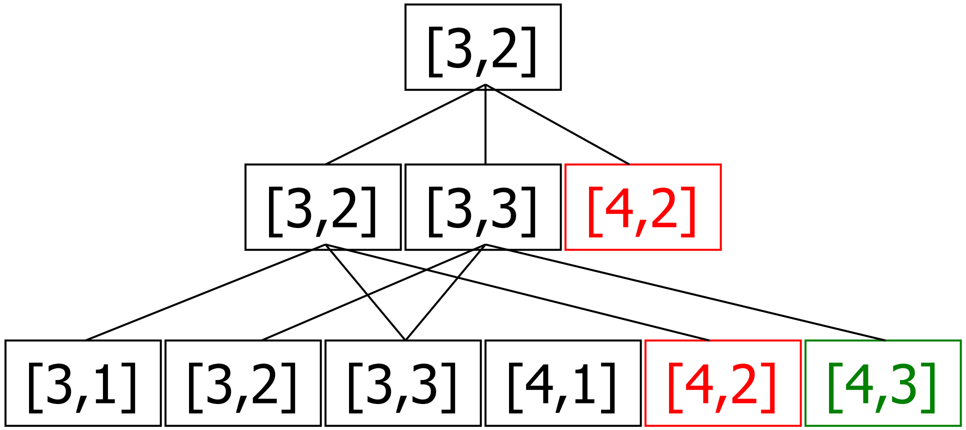

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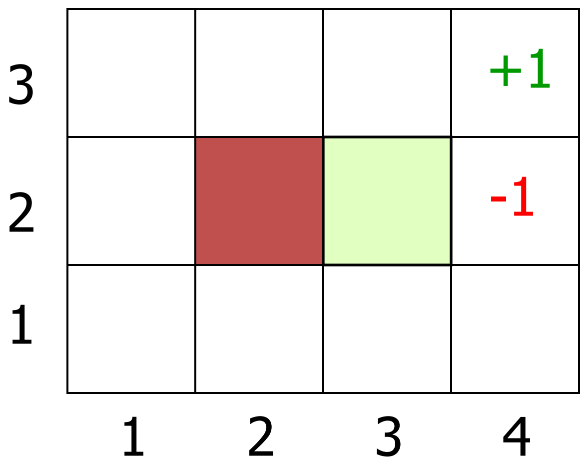
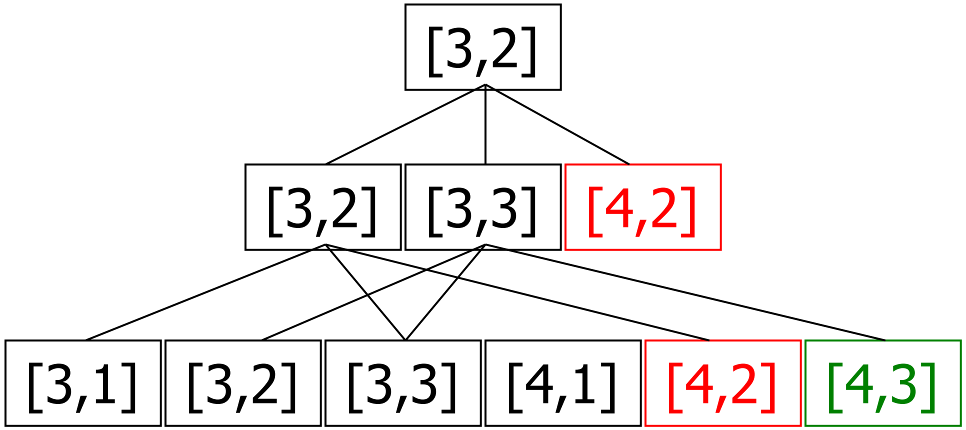
- **But is the optimal action sequence what we want to compute?**

Optimal Action Sequence





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only if the sequence is executed blindly!